Introduction to Logarithmic Functions

General Outcome: Develop algebraic and graphical reasoning through the study of relations.

SO4. Demonstrate an understanding of logarithms and the laws of logarithms. [C, CN, ME, R]

4.1 Express a logarithmic equation as an exponential equation and vice versa.
4.2 Determine the value of a logarithmic expression, such as log 2 8, without technology.
4.3 Develop the laws of logarithms, using numeric examples and the exponent laws.
4.4 Determine an equivalent expression for a logarithmic expression by applying the laws of logarithms.
4.5 Determine the approximate value of a logarithmic expression, such as log 2 9, with technology.\

SO5. Solve problems that involve exponential equations. [C, CN, PS, R, T]

5.2 Determine the solution of an exponential equation in which the bases are not powers of one another; e.g., $2x-1 = 3x+1$.
5.3 Solve problems that involve the application of exponential equations to loans, mortgages and investments.
5.4 Solve problems that involve logarithmic scales, such as the Richter scale and the pH scale.

SO6. Represent data, using exponential and logarithmic functions, to solve problems. [C, CN, PS, T, V]

6.1 Describe, orally and in written form, the characteristics of a logarithmic function by analyzing its graph.
6.2 Describe, orally and in written form, the characteristics of a logarithmic function by analyzing its equation.
6.3 Match equations in a given set to their corresponding graphs.
6.4 Graph data, and determine the logarithmic function that best approximates the data.
6.5 Interpret the graph of a logarithmic function that models a situation, and explain the reasoning.
6.6 Solve, using technology, a contextual problem that involves data that is best represented by graphs of logarithmic functions, and explain the reasoning.

Mathematical Processes

- **Connections [CN]** Students are expected to make *connections* among mathematical ideas, other concepts in mathematics, everyday experiences and other disciplines
- **Problem Solving [PS]** Students are expected to develop and apply new mathematical knowledge through *problem solving*
- **Reasoning [R]** Students are expected to develop mathematical *reasoning*

- **Visualization [V]** Students are expected to develop *visualization* skills to assist in processing information, making connections and solving problems
- **Technology [T]** Students are expected to select and use technology as a tool for learning and for solving problems
- **Mental Estimation [ME]** Students are expected to demonstrate fluency with mental mathematics and estimation.

Characteristics of Logarithmic Functions with Base 10 and Base *e*

Achievement Indicators:

4.1 Express a logarithmic equation as an exponential equation and vice versa.
6.1 Describe, orally and in written form, the characteristics of a logarithmic function by analyzing its graph.
6.2 Describe, orally and in written form, the characteristics of a logarithmic function by analyzing its equation.
6.3 Match equations in a given set to their corresponding graphs.

Lesson Links:

- Click here for a Notebook version of ERLC Lesson Link. Please use this lesson as a framework for your own teaching environment.
- Click here for a pdf version of the same ERLC Lesson Link.

Videos:

- Characteristics of Logarithmic Functions with Base 10 and Base e -- (Youtube Link)

Discovery Based Learning

1. Algebra 2: What is Log? -- TInspire Activity
   This lesson involves the one-to-one function f(x)=bx. In acknowledging the existence of its inverse, students will:
   - Use the domain and range of *f(x)* to determine the domain and range of *f-1(x)*.
   - Interpret the graph of $f^{1}(x)$ as the reflection of $f(x)$ across the line $y = x$.
   - Use this inverse relationship to write an equation for the graph of the inverse.
   - Recognize the logarithmic notation needed to define the inverse function.
   - Use the inputs and outputs of two inverse functions to complete a table. As a result, students will:
   - Solve simple logarithmic equations and verify solutions using the corresponding exponential equations.

2. Graph Logarithms -- TInspire Activity
   Students will investigate the graphs of the family of logarithm functions f(x)=loga(x), by changing the a-value over the interval 0 less than or equal to a less than or equal to 4. As a result, students will:
   - Infer why the conditions a>0 and a≠1 are necessary.
   - Determine how the value of a affects the increasing or decreasing behavior of the function.
   - Determine the x-intercept, domain, range, and asymptotes.
   - Describe the end behavior. NOTE: The time varies for this activity depending on whether students create the TI-Nspire document or use the pre-constructed .tns file.

Evaluating Logarithmic Expressions

Achievement Indicators:

4.1 Express a logarithmic equation as an exponential equation and vice versa.
4.2 Determine the value of a logarithmic expression, such as log 2 8, without technology.
4.5 Determine the approximate value of a logarithmic expression, such as log 2 9, with technology.
5.4 Solve problems that involve logarithmic scales, such as the Richter scale and the pH scale.

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Laws of Logarithms

Achievement Indicators:

4.3 Develop the laws of logarithms, using numeric examples and the exponent laws.
4.4 Determine an equivalent expression for a logarithmic expression by applying the laws of logarithms.

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Discovery Based Learning

1. Algebra 2: Properties of Logarithms (Nspired Learning Math Classroom)
   This lesson is based on observing counter-examples to logarithmic rules and involves comparing the values of several base 2 logarithmic expressions to discover which produce the same result. As a result, students will:
   - Generalize these rules for base *a*, where *a* is a real number, $a > 0$ and $a \neq 1$.
   - Compare these logarithmic properties to their exponential counterparts.

2. Investigating Laws of Logarithms Powerpoint

3. James Tanton -- Assessment Thoughts
   I think this could be used either as a prompt for Discovery or an Assessment for Learning Piece.

Question: Gordie thinks that the following is a valid log rule:

$$\log M \cdot \log N = \log(M + N)$$

He says that it turns multiplications into additions, which is indeed what logarithms do.

BUT HE IS NOT CORRECT!

What is the correct version of the log rule that Gordie is trying to write down? How would you advise him to think about the log rule so that it will be clear in his mind what the correct version should be?

Solving Exponential Equations Using Logarithm

Achievement Indicators:

4.5 Determine the approximate value of a logarithmic expression, such as log 2 9, with technology.
5.2 Determine the solution of an exponential equation in which the bases are not powers of one another; e.g., $2x-1 = 3x+1$.
5.3 Solve problems that involve the application of exponential equations to loans, mortgages and investments.

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Modelling Data Using Logarithmic Functions

Achievement Indicators:

6.4 Graph data, and determine the logarithmic function that best approximates the data.
6.5 Interpret the graph of a logarithmic function that models a situation, and explain the reasoning.
6.6 Solve, using technology, a contextual problem that involves data that is best represented by graphs of logarithmic functions, and explain the reasoning.

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