Introduction to Counting Methods

General Outcome: Develop critical thinking skills related to uncertainty.

SO4. Solve problems that involve the fundamental counting principle. [PS, R, V]

4.1 Represent and solve counting problems, using a graphic organizer.
4.2 Generalize, using inductive reasoning, the fundamental counting principle.
4.3 Identify and explain assumptions made in solving a counting problem.
4.4 Solve a contextual counting problem, using the fundamental counting principle, and explain the reasoning.

SO5. Solve problems that involve permutations. [ME, PS, R, T, V]
(It is intended that circular permutations not be included.)

5.1 Represent the number of arrangements of n elements taken n at a time, using factorial notation.
5.2 Determine, with or without technology, the value of a factorial.
5.3 Simplify a numeric or an algebraic fraction that contains factorials in both the numerator and denominator.
5.4 Solve an equation that involves factorials.
5.5 Determine the number of permutations of n elements taken r at a time.
5.6 Determine the number of permutations of n elements taken at a time where some elements are not distinct.
5.7 Explain, using examples, the effect on the total number of permutations of n elements when two or more elements are identical.
5.8 Generalize strategies for determining the number of permutations of n elements taken r at a time.
5.9 Solve a contextual problem that involves probability and permutations.

SO6. Solve problems that involve combinations. [ME, PS, R, T, V]

6.1 Explain, using examples, why order is or is not important when solving problems that involve permutations or combinations.
6.2 Determine the number of combinations of n elements taken r at a time.
6.3 Generalize strategies for determining the number of combinations of n elements taken r at a time.
6.4 Solve a contextual problem that involves combinations and probability.

Mathematical Processes

- **Connections [CN]** Students are expected to make *connections* among mathematical ideas, other concepts in mathematics, everyday experiences and other disciplines
- **Problem Solving [PS]** Students are expected to develop and apply new mathematical knowledge through *problem solving*
- **Reasoning [R]** Students are expected to develop mathematical reasoning
- **Visualization [V]** Students are expected to develop *visualization* skills to assist in processing information, making connections and solving problems.
- **Mental Estimation [ME]** Students are expected to demonstrate fluency with mental mathematics and estimation.
- **Technology [T]** Students are expected to select and use technology as a tool for learning and for solving problems

Lesson 1: Counting Principles

Achievement Indicators

4.1 Represent and solve counting problems, using a graphic organizer.
4.2 Generalize, using inductive reasoning, the fundamental counting principle.
4.3 Identify and explain assumptions made in solving a counting problem.
4.4 Solve a contextual counting problem, using the fundamental counting principle, and explain the reasoning.

Lesson Links:

- Click here for a Notebook version of ERLC Lesson Link. Please use this lesson as a framework for your own teaching environment.
- Click here for a pdf version of the same ERLC Lesson Link.

Additional Resources to Support this Lesson

- Absolute Value Workbook: Lesson 1: Fundamental Counting Principle -- Page 63
- John Tanton on Counting Principles – Click here

Videos:

1. Counting Principles -- (Youtube Link)
2. Visualizing the Fundamental Counting Principle (Vimeo Link)

Lesson 2: Introducing Permutations and Factorial Notation

Achievement Indicators

4.1 Represent and solve counting problems, using a graphic organizer.
4.4 Solve a contextual counting problem, using the fundamental counting principle, and explain the reasoning.
5.1 Represent the number of arrangements of n elements taken n at a time, using factorial notation.
5.2 Determine, with or without technology, the value of a factorial.
5.3 Simplify a numeric or an algebraic fraction that contains factorials in both the numerator and denominator.
5.4 Solve an equation that involves factorials.

Discovery Based Learning

1. Algebra 2: Permutations -- TInspire Activity [T]
   Students are led through the development of the formula for finding *n* objects taken *n* at a time and then *n* objects taken *r* at a time.

Lesson Links:

- Click here for ERLC Lesson Link. Please use this lesson as a framework for your own teaching environment.
- Click here for a pdf version of the same ERLC Lesson Link.

Videos:

- Permutations and Factorial Notation -- (Youtube Link)

Lesson 3: Permutations When All Objects are Distinguishable

Achievement Indicators:

4.1 Represent and solve counting problems, using a graphic organizer.
4.4 Solve a contextual counting problem, using the fundamental counting principle, and explain the reasoning.
5.2 Determine, with or without technology, the value of a factorial.

5.3 Simplify a numeric or an algebraic fraction that contains factorials in both the numerator and denominator.
5.4 Solve an equation that involves factorials.
5.5 Determine the number of permutations of n elements taken r at a time.
5.8 Generalize strategies for determining the number of permutations of n elements taken r at a time.

Lesson Links:

- Click here for ERLC Lesson Link. Please use this lesson as a framework for your own teaching environment.
- Click here for a pdf version of the same ERLC Lesson Link.

Videos:

- Permutations When All Objects Are Distinguishable -- (Youtube Link)

Lesson 4: Permutations When Objects are Identical

Achievement Indicators:

4.1 Represent and solve counting problems, using a graphic organizer.
4.3 Identify and explain assumptions made in solving a counting problem.
5.2 Determine, with or without technology, the value of a factorial.
5.3 Simplify a numeric or an algebraic fraction that contains factorials in both the numerator and denominator.
5.6 Determine the number of permutations of n elements taken n at a time where some elements are not distinct.
5.7 Explain, using examples, the effect on the total number of permutations of n elements when two or more elements are identical.

Lesson Links:

- Click here for ERLC Lesson Link. Please use this lesson as a framework for your own teaching environment.
- Click here for a pdf version of the same ERLC Lesson Link.

Videos:

- Permutations When Objects Are Identical -- (Youtube Link)
- Route Problems -- (Youtube Link)

Lesson 5: Exploring Combinations

Achievement Indicators:

4.1 Represent and solve counting problems, using a graphic organizer.
5.2 Determine, with or without technology, the value of a factorial.
5.5 Determine the number of permutations of n elements taken r at a time.
5.8 Generalize strategies for determining the number of permutations of n elements taken r at a time.
6.1 Explain, using examples, why order is or is not important when solving problems that involve permutations or combinations.
6.2 Determine the number of combinations of n elements taken r at a time.
6.3 Generalize strategies for determining the number of combinations of n elements taken r at a time.

Discovery Based Learning

1. Algebra 2: Permutations & Combinations -- TInspire Activity [T]
   Students explore permutations and combinations by arranging letters when order does and does not make a difference.

Lesson Links:

- Click here for ERLC Lesson Link. Please use this lesson as a framework for your own teaching environment.
- Click here for a pdf version of the same ERLC Lesson Link.

Videos:

- Exploring Combinations -- (Youtube Link)

Lesson 6: Combinations

Achievement Indicators:

4.1 Represent and solve counting problems, using a graphic organizer.
5.2 Determine, with or without technology, the value of a factorial.

5.5 Determine the number of permutations of n elements taken r at a time.

5.8 Generalize strategies for determining the number of permutations of n elements taken r at a time.

6.1 Explain, using examples, why order is or is not important when solving problems that involve permutations or combinations.
6.2 Determine the number of combinations of n elements taken r at a time.
6.3 Generalize strategies for determining the number of combinations of n elements taken r at a time.

Lesson Links:

- Click here for ERLC Lesson Link. Please use this lesson as a framework for your own teaching environment.
- Click here for a pdf version of the same ERLC Lesson Link.

Videos:

- Combinations -- (Youtube Link)

Lesson 7: Solving Counting Problems

Achievement Indicators:

4.1 Represent and solve counting problems, using a graphic organizer.
4.3 Identify and explain assumptions made in solving a counting problem.
4.4 Solve a contextual counting problem, using the fundamental counting principle, and explain the reasoning
5.2 Determine, with or without technology, the value of a factorial.
5.3 Simplify a numeric or an algebraic fraction that contains factorials in both the numerator and denominator.
5.5 Determine the number of permutations of n elements taken r at a time.
6.1 Explain, using examples, why order is or is not important when solving problems that involve permutations or combinations.
6.2 Determine the number of combinations of n elements taken r at a time.

Lesson Links:

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- Click here for a pdf version of the same ERLC Lesson Link.

Assessment For Learning Ideas

1. **Row Game: Permutations & Combinations --** (Download: RG-Permutations & Combinations.docx)
   Person A solves the problems in column A, while Person B solves the problems in column B. As you finish each problem, check your answer with the other person. The answers should be the same! If they are not, work together to find your mistake.

Videos

- Solving Counting Problems -- (Youtube Link)\

Unit Shared Resources

Unit Lessons

- Counting Methods Notebook Lessons -- Shared by Allen
- Counting Methods Teacher Notebook Lessons -- Shared by Boulanger
    - Counting Methods Student Word Lessons that accompany the Teacher Notebook Lessons -- Shared by Boulanger
- Classifying Word Problems Notebook Lesson - Shared by Boulanger
    - Classifying Word Problems Word Lesson -- Shared by Boulanger

Unit Review

- Counting Methods Unit Review -- Diploma Style -- Shared by Fern
- Outcome Based Review -- Shared by Fowler
- Counting Methods Review Class (Notebook File) -- Shared by Boulanger
- Jeopardy Review Game (PowerPoint) - Shared by McInnes