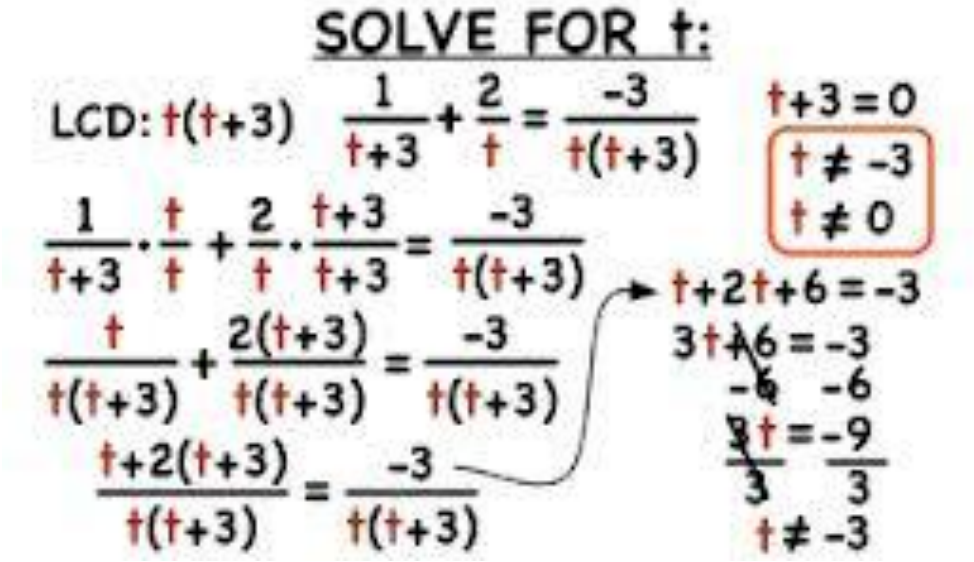
9.3 CONNECTING GRAPHS AND RATIONAL EQUATIONS

The solutions for an equation are also referred to as the roots of the equation.

The roots of an equation are related to the zeros of the related function.

The roots of an equation are related to the x-intercepts of the graph of the related function.



Solving Rational Equations

Determine if each of the following statements are true or false:

1. $x = 2$ is a solution to $\dfrac{3x+6}{x-1} = \dfrac{x^2+8}{x^2-3}$

True ✓ False ☐

2. $x = -3$ is a solution to $\dfrac{-2x+1}{1-x} = \dfrac{3x-2}{4}$

True ☐ False ✓

3. The equation $\dfrac{x^2-10}{x+1} = \dfrac{-3x}{x+1}$ has exactly 2 solutions.

True ✓ False ☐

Solving Equations with Rational Expressions - Algebraically

Solve: $\dfrac{3x-2}{x-2} = \dfrac{6}{x^2-4} + 1$

$$\frac{3x-2}{x-2} = \frac{6}{(x+2)(x-2)} + 1$$

LCD: $(x + 2)(x - 2)$
NPVs are ± 2

Multiply all terms by the LCD

$$\frac{(3x-2)(x+2)(x-2)}{(x-2)} = \frac{6(x+2)(x-2)}{(x+2)(x-2)} + 1(x+2)(x-2)$$

$$(3x-2)(x+2) = 6 + (x+2)(x-2)$$

$$3x^2 + 6x - 2x - 4 = 6 + x^2 - 2x + 2x - 4$$

$$3x^2 + 4x - 4 = x^2 + 2$$

$$3x^2 + 4x - 4 = x^2 + 2$$

$$2x^2 + 4x - 6 = 0$$

$$2\left(x^2 + 2x - 3\right) = 0$$

$$2(x+3)(x-1) = 0$$

$$x + 3 = 0 \quad \text{or} \quad x - 1 = 0$$

$$x = -3 \quad \text{or} \quad x = 1$$

Verify your solutions by substitution!

$$\frac{3x-2}{x-2} = \frac{6}{x^2-4} + 1$$

$x = -3$

$$\frac{3(-3)-2}{(-3)-2} = \frac{6}{(-3)^2-4} + 1$$

$$\frac{-11}{-5} = \frac{6}{5} + \frac{5}{5}$$

$$\frac{11}{5} = \frac{11}{5}$$

$x = 1$

$$\frac{3(1)-2}{(1)-2} = \frac{6}{(1)^2-4} + 1$$

$$\frac{1}{-1} = \frac{6}{-3} + 1$$

$$-1 = -1$$

Solving Equations with Rational Expressions - Graphically

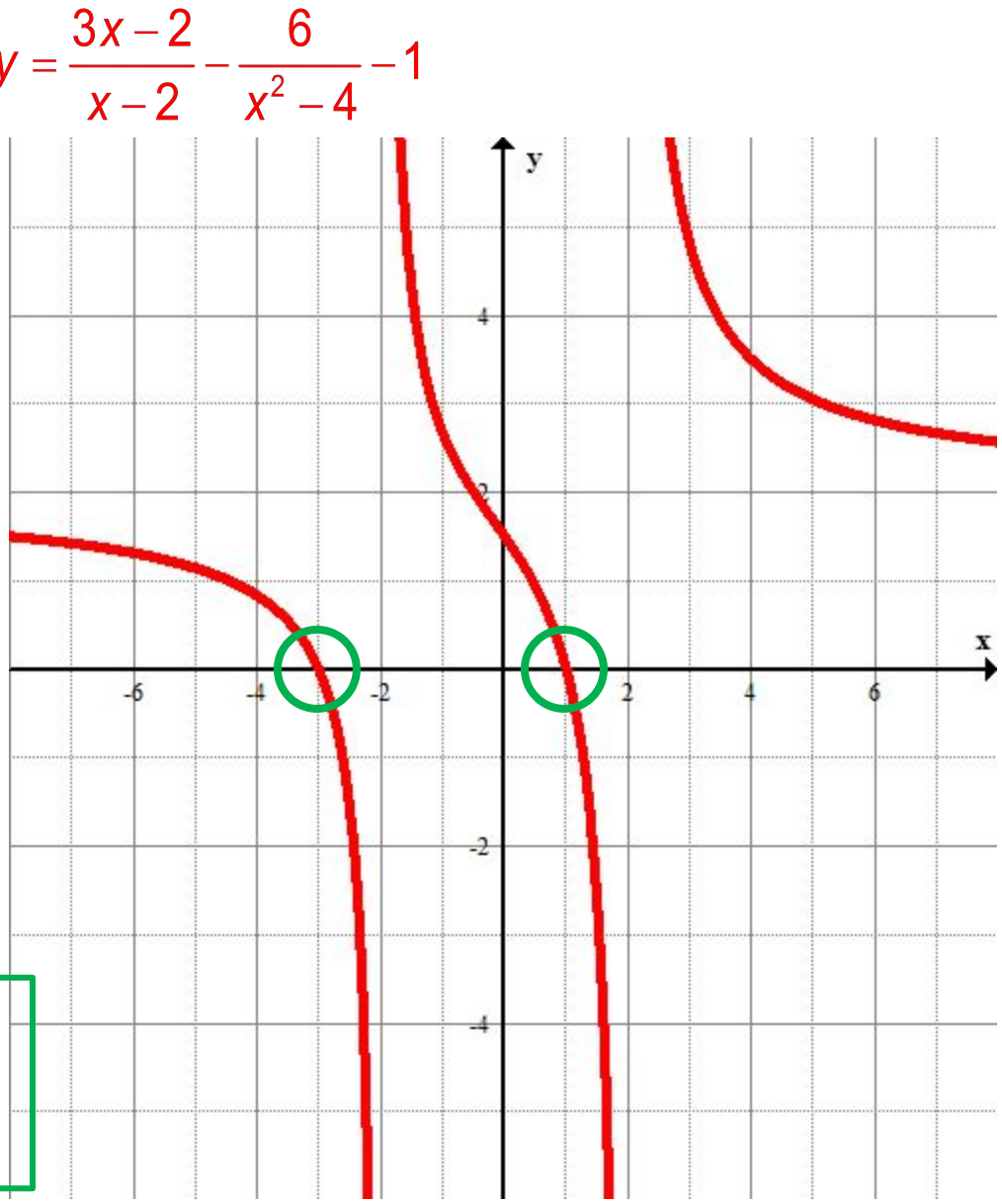
$$\frac{3x-2}{x-2} = \frac{6}{x^2-4} + 1$$

Rewrite the equation equating it to 0.

$$\frac{3x-2}{x-2} - \frac{6}{x^2-4} - 1 = 0$$

Graph the related function.

$$y = \frac{3x-2}{x-2} - \frac{6}{x^2-4} - 1$$



The roots of the equation are the *x*-intercepts of the graph: *x* = –3 and *x* = 1.

Solving Equations w/ Rational Expressions - Algebraically

Solve: $x - \frac{6}{x+3} = \frac{2x}{x+3} + 2$

NPV is –3

LCD: $x + 3$

$$(x+3)x - (x+3)\frac{6}{x+3} = (x+3)\frac{2x}{x+3} + (x+3)2$$

$$x^2 + 3x - 6 = 2x + 2x + 6$$

$$x^2 + 3x - 6 = 4x + 6$$

$$x^2 - x - 12 = 0$$

$$(x+3)(x-4) = 0$$

$$x + 3 = 0 \qquad x - 4 = 0$$

$$\cancel{x = -3} \qquad x = 4$$

Solving Equations Graphically Method 2

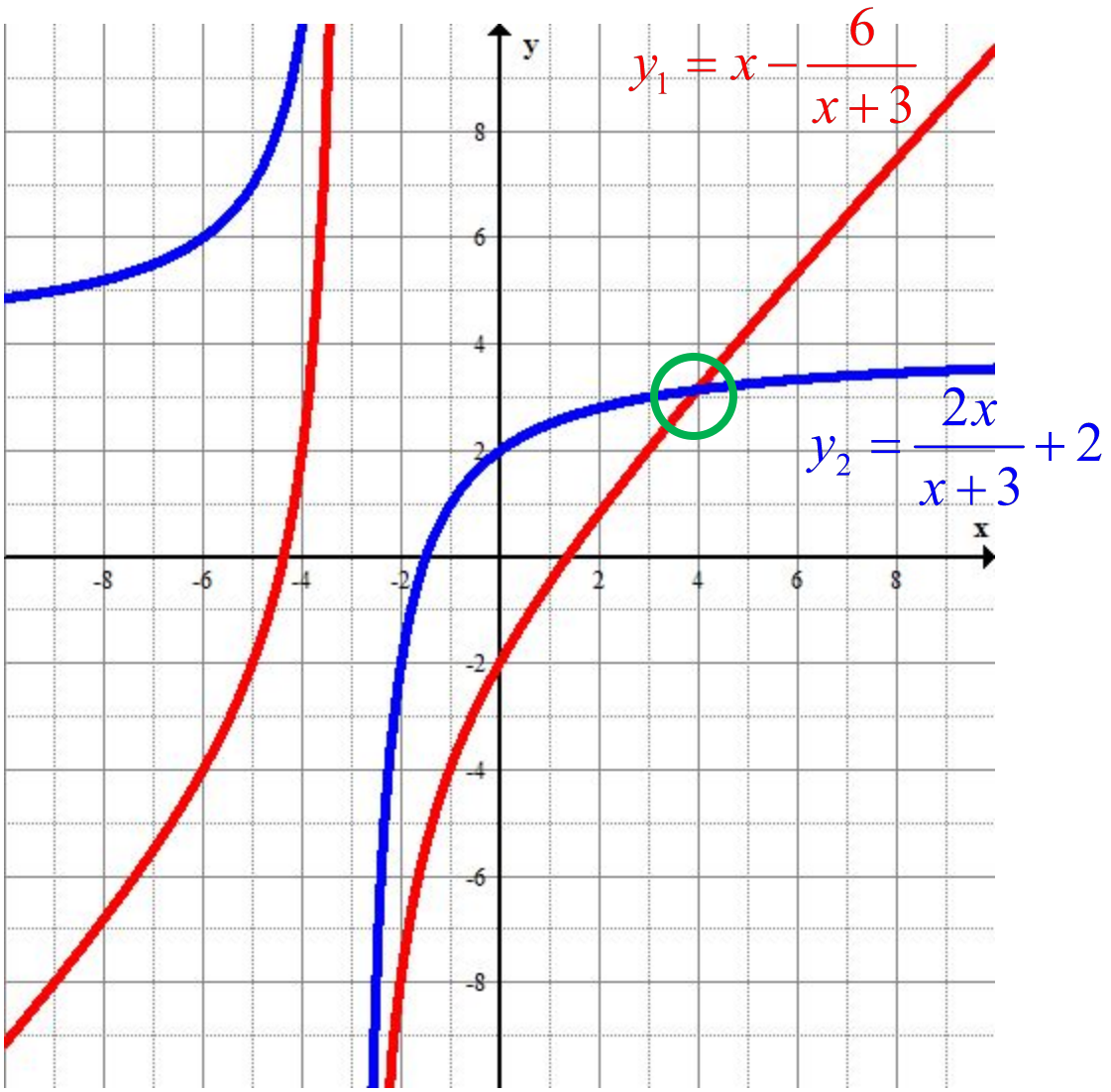
Graph the expression on each side of the equals sign and then locate the points of intersection.

Graph:

$$y_1 = x - \frac{6}{x+3}$$

$$y_2 = \frac{2x}{x+3} + 2$$

The solution is where the graphs intersect at $x = 4$.



Solving Equations with Rational Expressions - Your Turn

Solve: $\dfrac{x+1}{3x-6} = \dfrac{5x}{6} + \dfrac{1}{x-2}$

LCD: $6(x-2)$
NPV is 2

$$6(x-2)\frac{x+1}{3(x-2)} = 6(x-2)\frac{5x}{6} + 6(x-2)\frac{1}{x-2}$$

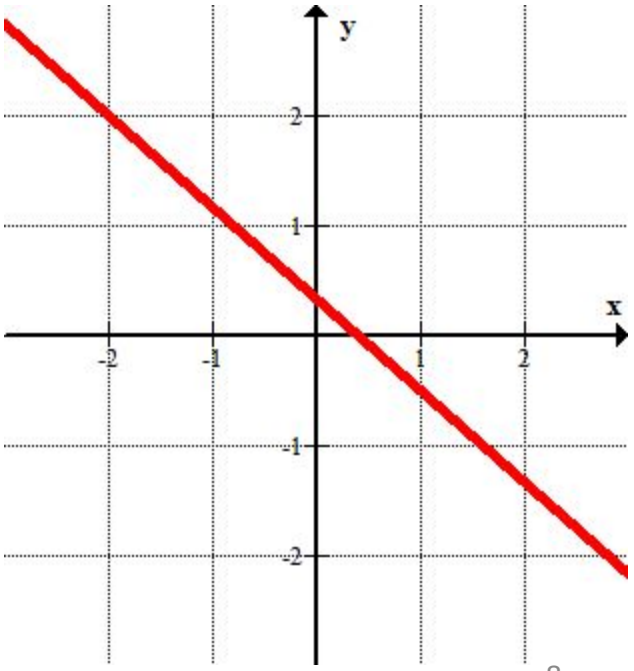
$$2(x+1) = 5x(x-2) + 1\cdot 6$$

$$2x+2 = 5x^2 - 10x + 6$$

$$0 = 5x^2 - 12x + 4$$

$$0 = (5x-2)(x-2)$$

$x = \dfrac{2}{5}$ $\quad$ ~~$x = 2$~~



Solve a Problem Using Rational Equations

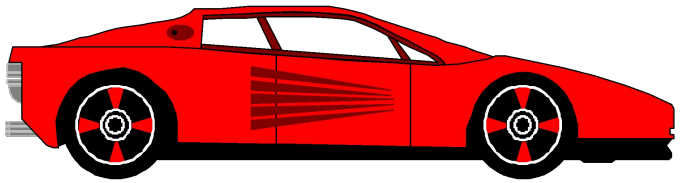
A car travels 500 miles in the same time that a train travels 300 miles. The speed of the car is 30 miles per hour faster than the speed of the train. Determine the speed of the car and the train.

Let r = speed of the train

$r + 30$ = speed of the car

Car's Time = Train's Time

$$t = \frac{d}{r}$$



$$t = \frac{500}{r+30}$$

$$t = \frac{300}{r}$$

Solve a Problem Using Rational Equations

LCD: $r(r + 30)$
NPVs are 0, –30

$$\frac{500}{r+30} = \frac{300}{r}$$

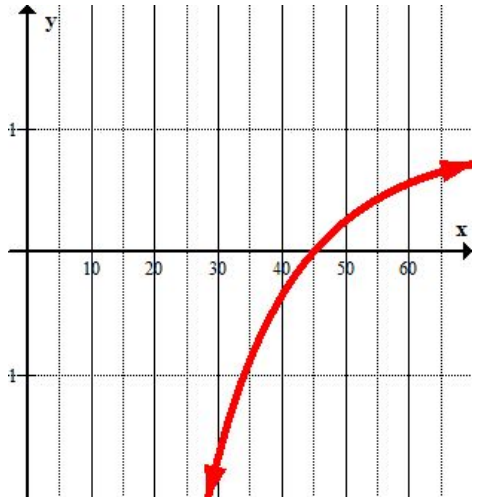
$$r(r+30)\frac{500}{r+30} = \frac{300}{r}r(r+30)$$

$$500r = 300(r+30)$$

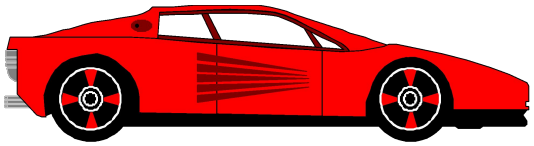
$$500r = 300r + 9000$$

$$200r = 9000$$

$$r = 45$$



The car travels at a speed of 75 mph



The train travels at a speed of 45 mph

Assignment

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