YACK IN THE BOX

- Fractions
- Equivalence
- Writing equations

Getting Ready

What You'll Need

Cuisenaire Rods, 1 set per pair

Small shoe boxes or tissue boxes

1-centimeter grid paper, page 120

Activity Master, page 102

Overview

Students use a combination of two Cuisenaire Rods to form a longer rod segment. Assuming that this rod represents one whole unit, students form addition and subtraction sentences involving the fractional lengths of the other rods. In this activity, students have the opportunity to:

- investigate fractional parts of the whole
- create symbolic equations
- find fractional sums and differences
- represent fractions with equivalent expressions

Other *Super Source* activities that explore these and related concepts are:

Playground Equipment, page 34

Foreign Currency Exchange, page 39

The Activity

On Their Own (Part 1)

Chen is creating a new game called "Yack in the Box," and he needs assistance in finding the values of the playing pieces. How can he determine the fractional values of his game pieces?

- Work with a partner. A "yack" is a new rod whose length consists of 1 yellow Cuisenaire Rod and 1 black Cuisenaire Rod attached end to end.
- Using Cuisenaire Rods, make all one-color combinations that will match the length of the yack.
- Assume that the length of the yack represents one whole unit. For each of the one-color combinations, find the fractional part of a single rod in relation to the whole yack. Record the color of each rod and its fractional value.
- Now find the fractional values of each of the remaining rods with the yack representing 1 whole unit. Add your findings to the data already collected and arrange your data in increasing order of value.
- Look for patterns and relationships in the data. Be ready to explain your findings.

Thinking and Sharing

Invite students to share and explain their findings. Create a chart naming each rod color and its fractional part of the whole.

Use prompts like these to promote class discussion:

- Which one-color rod combinations matched the length of the yack?
- How did you determine the fractional part that each of these color rods represented in relation to the whole?
- How did you determine the fractional values of the remaining rods?
- How did you determine the order of the fractions?
- What patterns or relationships did you discover?

On Their Own (Part 2)

What if... ***Chen is ready for you to play a game of "Yack in the Box"? Can you determine fractional relationships among the rods to help you win the game?***

- Work in pairs. Place a set of Cuisenaire Rods in a small box. Decide who will go first. Player A randomly selects 3 rods from the box.
- On a sheet of paper, Player A writes two addition sentences about the 3 rods. One equation should relate the colors of the rods and the other equation should relate their fractional values. Fractional values are to be expressed in terms of yacks. For example, $r + g + p = e$; $1/6$ yack + $1/4$ yack + $1/3$ yack = $3/4$ yack.
- Then Player A selects 2 of the 3 rods, and writes two subtraction sentences about them. One equation should relate the colors of the rods and the other equation should relate their fractional values. As before, fractional values should be expressed in terms of yacks. For example, $p - g = w$; $1/3$ yack – $1/4$ yack = $1/12$ yack.
- Player B checks Player A's equation sheet. If all statements are correct, Player A earns 1 point. If a mistake(s) is found, Player B can make the correction(s), and then he or she receives the point.
- After returning the 3 rods to the box, Player B selects 3 rods from the box, and he or she repeats the activity.
- Play continues by alternately drawing rods, writing the sets of equations, and checking results. The first player to earn 8 points is the winner.
- Be ready to discuss the results of your game.

Thinking and Sharing

Invite pairs to discuss the results of their game. Encourage students to justify their equations and corrections.

Use prompts like these to promote class discussion:

- Were some addition statements harder to generate than others? Were some easier? Which ones and why?
- How did you select which two rods to use in the subtraction activity?
- What was difficult about writing the subtraction equations? What was easy?
- What mistakes, if any, do you think you could have avoided?

For Their Portfolio

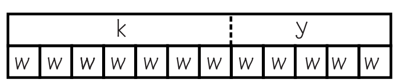
Write a letter to Chen explaining the processes used when adding fractions and subtracting fractions. Include an explanation about the skills you used while playing “Yack in the Box.”

Teacher Talk

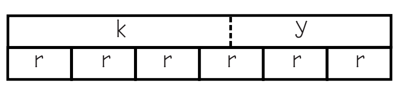
Where’s the Mathematics?

If you combine the yellow and the black Cuisenaire Rods, you will get a new rod called a “yack,” whose length equals the sum of the lengths of its two components. When students search for one-color rod combinations that will match the length of the yack, they soon discover that only the white, red, light green, purple, and dark green rods will work. By counting how many of each rod are needed to make one-color yacks, students can determine the fractional part of the whole represented by each rod.

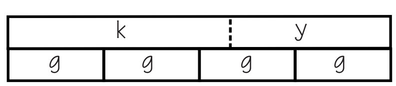
{“yack”}



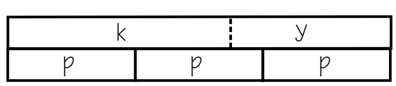
12 w = 1 yack ∴ w = $\frac{1}{12}$ yack



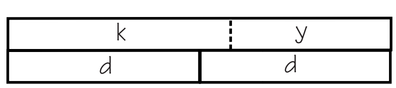
6 r = 1 yack ∴ r = $\frac{1}{6}$ yack



4 g = 1 yack ∴ g = $\frac{1}{4}$ yack

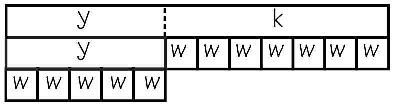


3 p = 1 yack ∴ p = $\frac{1}{3}$ yack

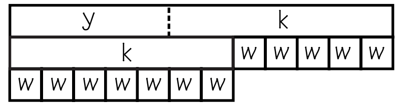


2 d = 1 yack ∴ d = $\frac{1}{2}$ yack

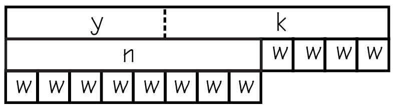
To determine what fractional part of the whole the remaining yellow, black, brown, blue, and orange rods are, students need to determine that the fractional value of the white rod is $\frac{1}{12}$ of the "yack" rod. A given number of these white rods can be attached end to end to match the length of each of the other rods. By matching the lengths, students can find the following information:



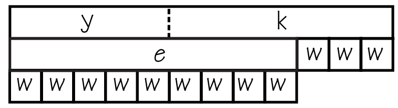
yellow = $\frac{5}{12}$ yack



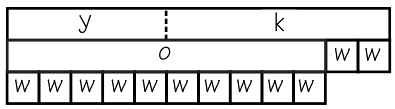
black = $\frac{7}{12}$ yack



brown = $\frac{8}{12}$ or $\frac{2}{3}$ yack



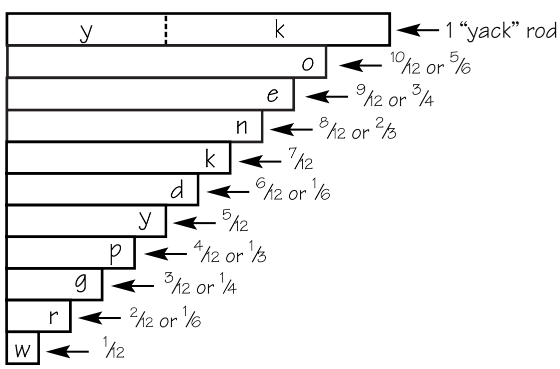
blue = $\frac{9}{12}$ or $\frac{3}{4}$ yack



orange = $\frac{10}{12}$ or $\frac{5}{6}$ yack

By comparing the brown rod to the purple rod, students may notice that 2 purple rods are needed to equal the length of the brown rod. In other words, the brown rod also represents $2(\frac{1}{3})$, or $\frac{2}{3}$ yack. Comparing the blue rod to the light green rod, students notice that 3 light green rods are needed to equal the length of the blue rod. Thus, the blue rod also represents $3(\frac{1}{4})$, or $\frac{3}{4}$ yack. Likewise, the orange rod is the same length as 5 red rods and must also represent the fraction $5(\frac{1}{6})$, or $\frac{5}{6}$ yack.

Before they start to play *Yack in the Box,* in Part 2, students might find it convenient to draw a diagram similar to the one below, in which increasing lengths are matched up with their fractional values.

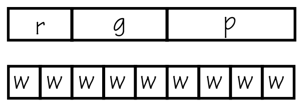


Playing *Yack in the Box* gives students the opportunity to work with concrete models of fractions and then translate these models into symbolic equations. The problems generated in the game provide practice in adding and subtracting fractions. The results of these arithmetic problems may involve proper fractions, whole numbers, improper fractions, or mixed numbers. Using rods to visualize the subtraction operation, in which the shorter rod is taken away from the longer rod, will help students avoid the problem of subtracting the larger fraction from the smaller fraction and generating a negative number.

Many students will place the three rods drawn from the box end to end and compare their total length to each of the rods in the yack family to find one that matches. If a rod is found, its fractional value represents the sum of the three rods.

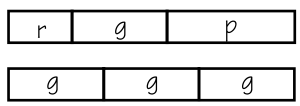
If the colors of the three rods are the same, students can find their sum by adding their numerators and keeping the same denominator. For example, when three red rods are added together, the resulting sum of $\frac{1}{6} + \frac{1}{6} + \frac{1}{6}$ becomes $\frac{3}{6}$.

If the colors of the three rods are not the same, however, the 3 rods must be exchanged for equivalent length rods all using the same color. For example, when red, light green, and purple rods are drawn from the box, the red rod can be thought of as 2 whites ($\frac{2}{12}$), the light green rod as 3 whites ($\frac{3}{12}$), and the purple rod as 4 whites ($\frac{4}{12}$). Once the colors are the same, students can add the fractions to get 9 whites ($\frac{9}{12}$). Some students may then choose to write the equations using the fractional name for each rod expressed in lowest terms, as shown below.



$$\frac{2}{12} + \frac{3}{12} + \frac{4}{12} = \frac{9}{12}$$
$$\frac{1}{6} + \frac{1}{4} + \frac{1}{3} = \frac{9}{12} \text{ or } \frac{3}{4}$$

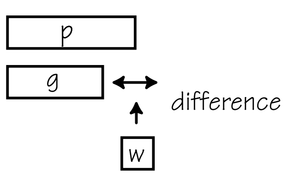
Other students who are working with the same red, light green, and purple rods may continue to search until they find the minimum number of one-color rods that will match the length of these three rods. The sum can then be written in terms of light green rods.



$$\frac{2}{12} + \frac{3}{12} + \frac{4}{12} = \frac{9}{12}$$
$$\frac{3}{12} + \frac{3}{12} + \frac{3}{12} = \frac{9}{12}$$

Many different number sentences or equations can be written to express the results of adding the red, light green, and purple rods together. Talking about how these sentences differ, but are really equivalent, will help students see the role of equivalent fractions in forming sums. Students may also generate different equations by using the Commutative Property of Addition to rearrange the order of the fractions being added.

Once students have selected the two rods to be subtracted, they can visually represent their difference by laying the two rods side by side. In the case where the two rods are identical, their difference will equal zero. If the two rods are of different lengths, the distance from the end of the shorter rod to the end of the longer rod represents the difference between the rods. The example below shows the problem of subtracting the light green rod from the purple rod and the appropriate symbolic and numeric equations.



symbolic	numeric
$\begin{array}{r} p \\ -\,g \\ \hline w \end{array}$	$\begin{array}{rcl} \frac{1}{3} & \text{or} & \frac{4}{12} \\ -\,\frac{1}{4} & \text{or} & \frac{3}{12} \\ \hline & \frac{1}{12} & \end{array}$

or $p - g = w$

$\frac{1}{3}$ yack $-$ $\frac{1}{4}$ yack $= \frac{4}{12}$ yack $-$ $\frac{3}{12}$ yack $= \frac{1}{12}$ yack

Using letters to write equations based on the colors of Cuisenaire Rods serves as an introduction to later work with variables and algebraic equations.