Predicting the Future

Focusing Question: How many blocks are in Figure 100?

Background Information

Scientists look for numerical patterns in everything from planetary arrangement to the frequency of volcanic eruptions in the hopes of being able to make predictions about future behavior. The block pictures in this assignment aren't that dramatic, but their patterns are more consistent and easier to predict. The trick is, can you do this without knowing the answer to the previous one? For example, do you have to know how many blocks are in Figure 99 to find an answer to Figure 100? Counting all the way up is one possibility, but there are shorter ways.

Questions

1. For each set of diagrams, draw Figures 4 and 5, then figure out the # of blocks in Figure 100. Use any method, but explain how you get your answer.

2. Write an expression that shows the relationship between the figure # (x) and the number of blocks (y). Substitute x values to show that it works. Use colors to show the relationship between your expression and the diagrams.

3. Graph each relationship (use x values from 0-5). Which graphs form a perfectly straight line?

4. Why are some graphs straight lines and others curved?

Bonus

- Work in reverse: How would you draw the growth of a pattern for 3x + 1? Is there more than one way?
- What happens if you think of all the shapes as three-dimensional and give each stage of growth a new layer?
- Invent some patterns of your own. What happens if you design patterns that grow in three dimensions?
- Use a spreadsheet to graph each of the relations for x values from –10 to +10. What patterns do you notice in the graphs?
- Invent some of your own relations, predict what their graphs will look like, then test your predictions! (For example, what happens if you use 3x instead of 2x? What if you use x^3 instead of x^2? What if you add a constant of 3 instead of 4?)
- How might you draw a picture to represent x=0 or x=-1? y=0?

1		2			3				4			
x	**y**											
1												
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5												
100												

1		2			3				4		
x	**y**										
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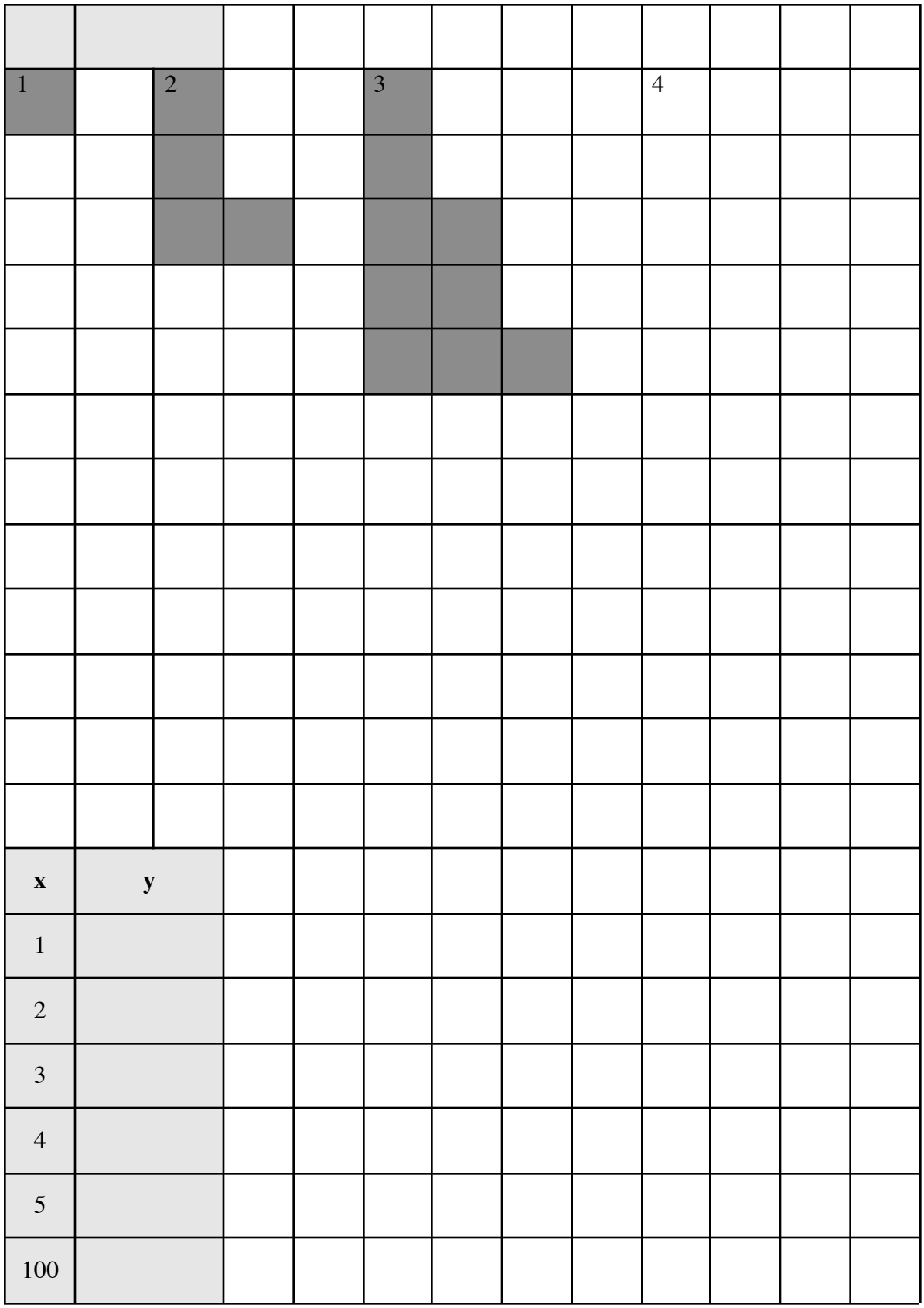
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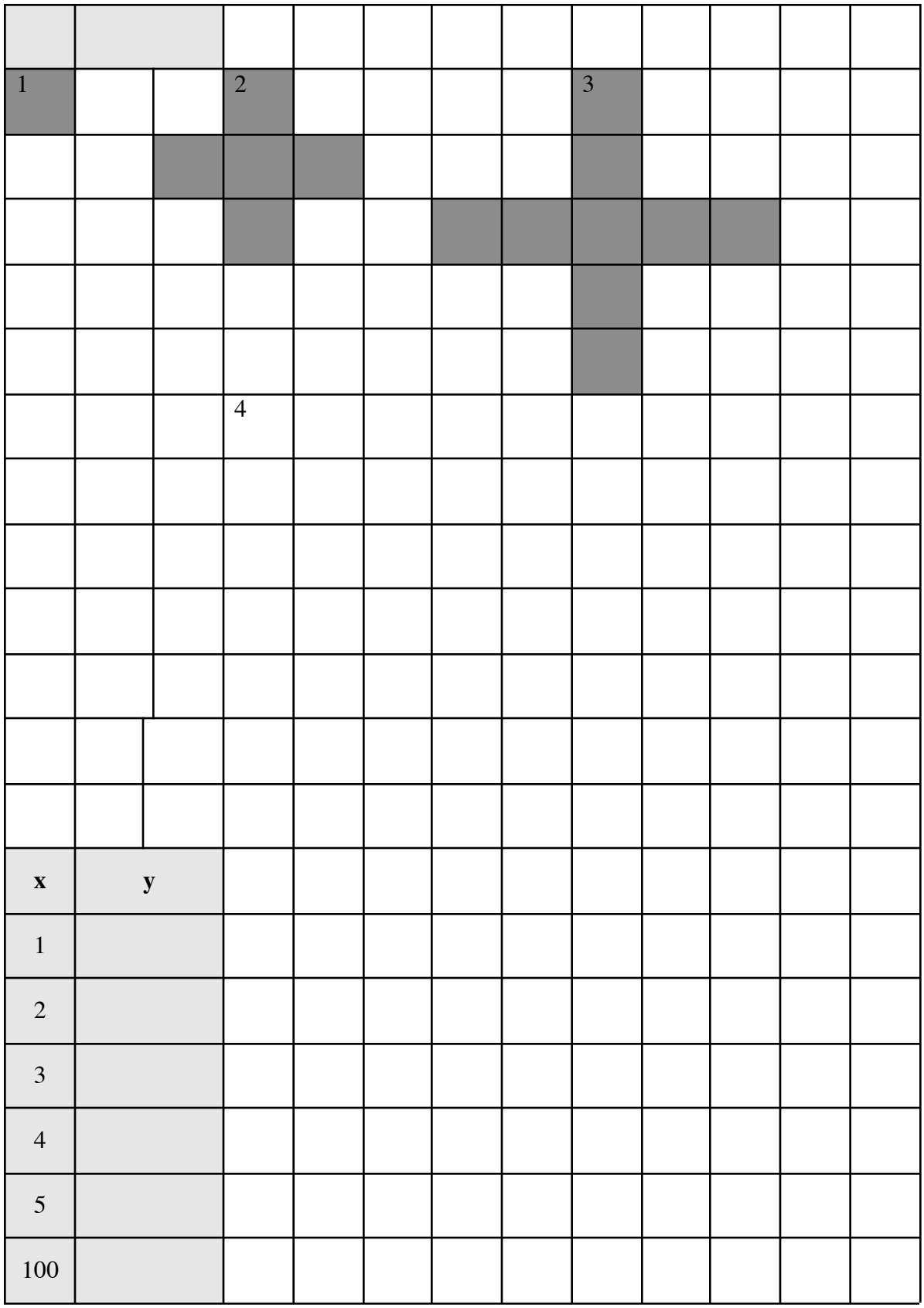

x	y
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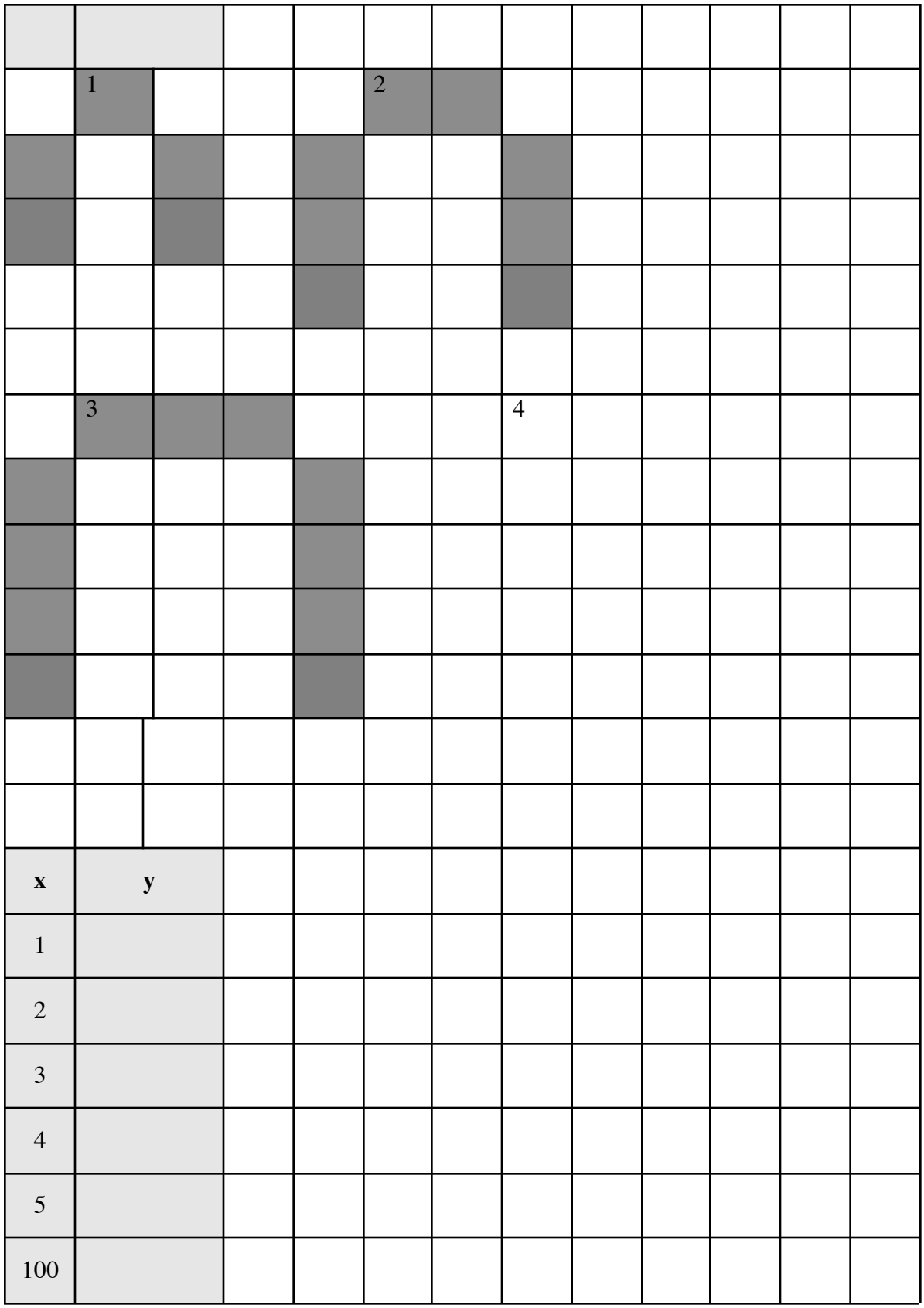
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x	**y**										
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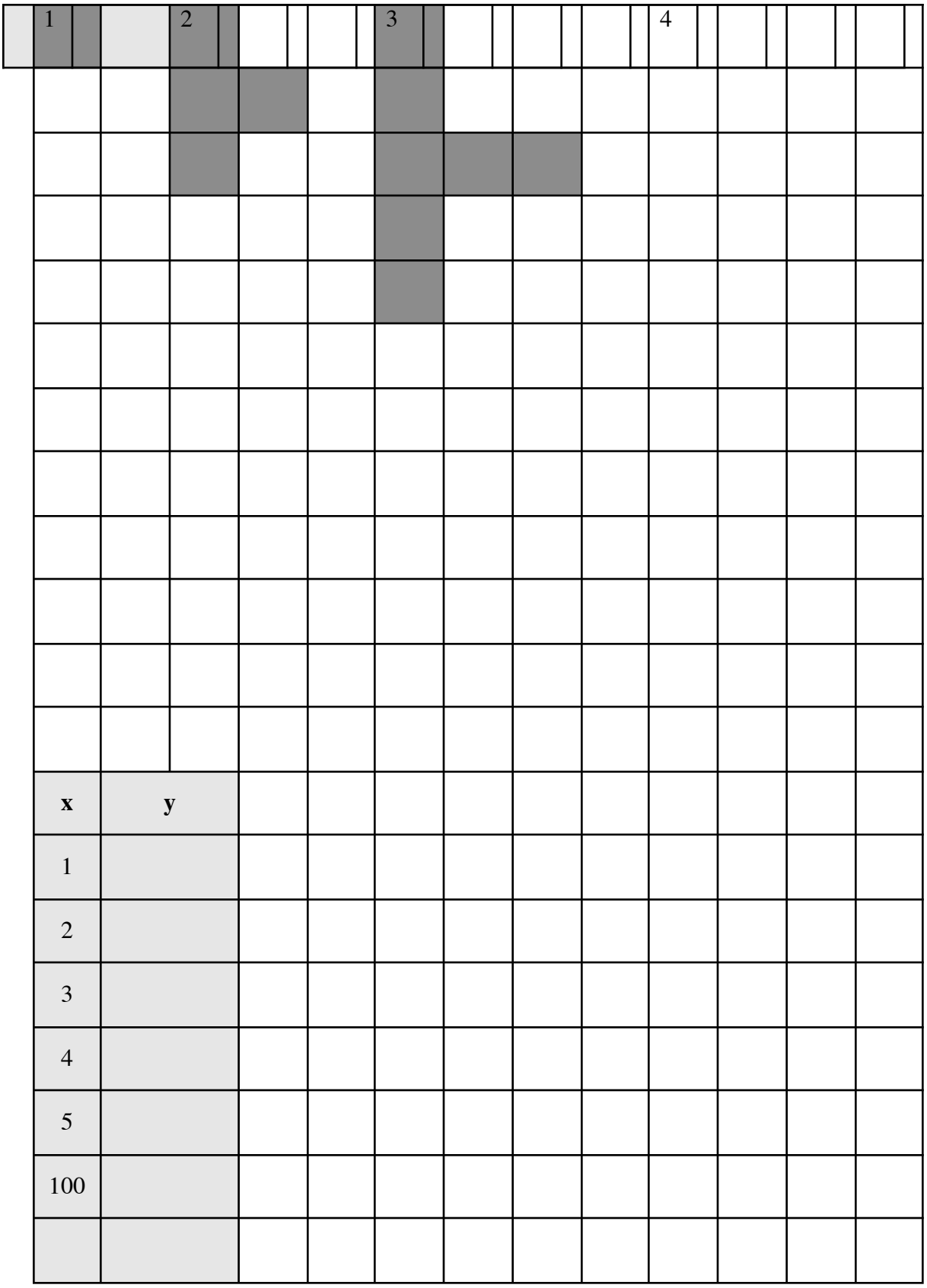

Also see “Alonzo’s staircase” (4 converging 3-D staircases) in Boaler, 2008, p. 168

1			2			3			4			
x	**y**											
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x	**y**											
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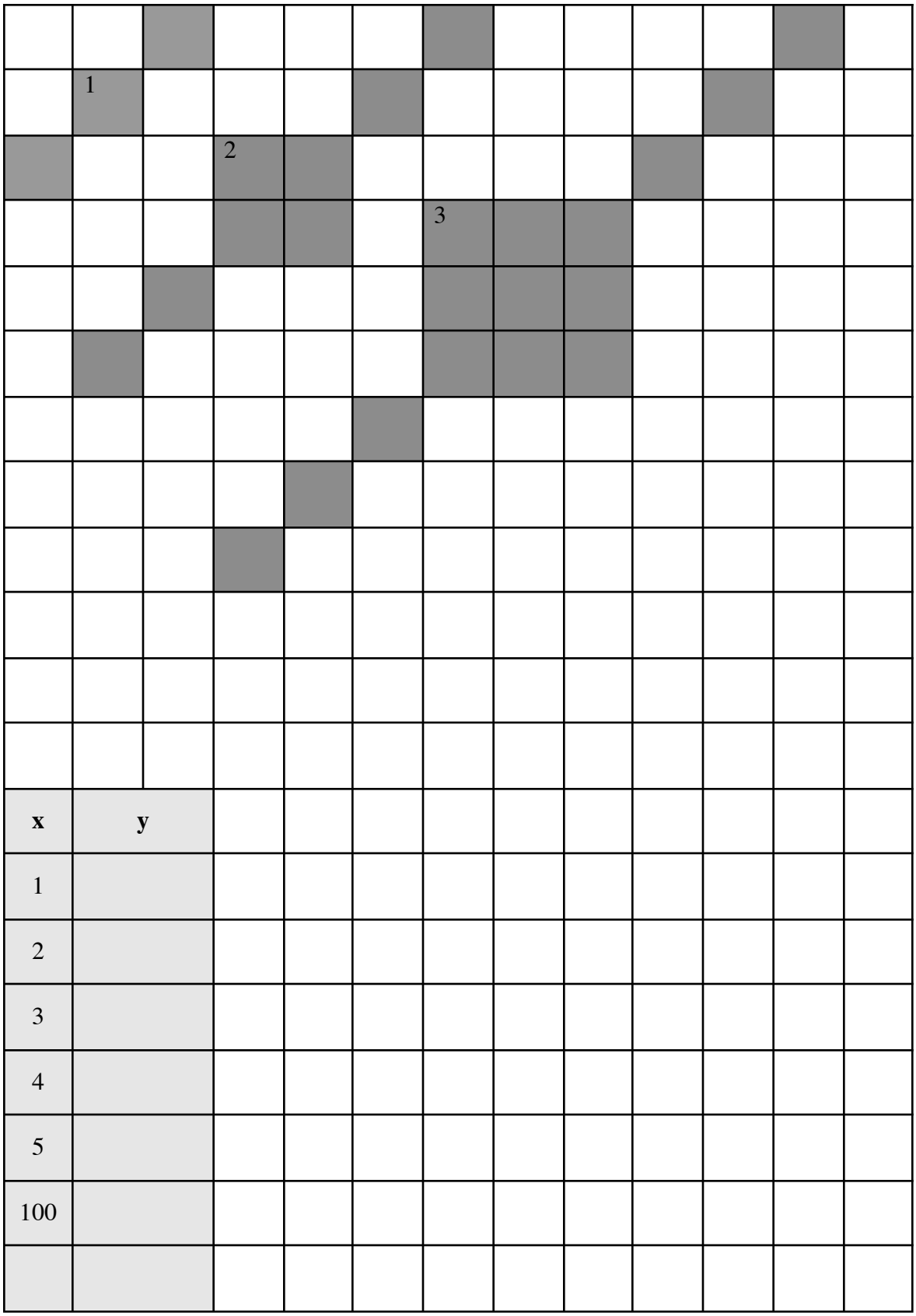

x	y
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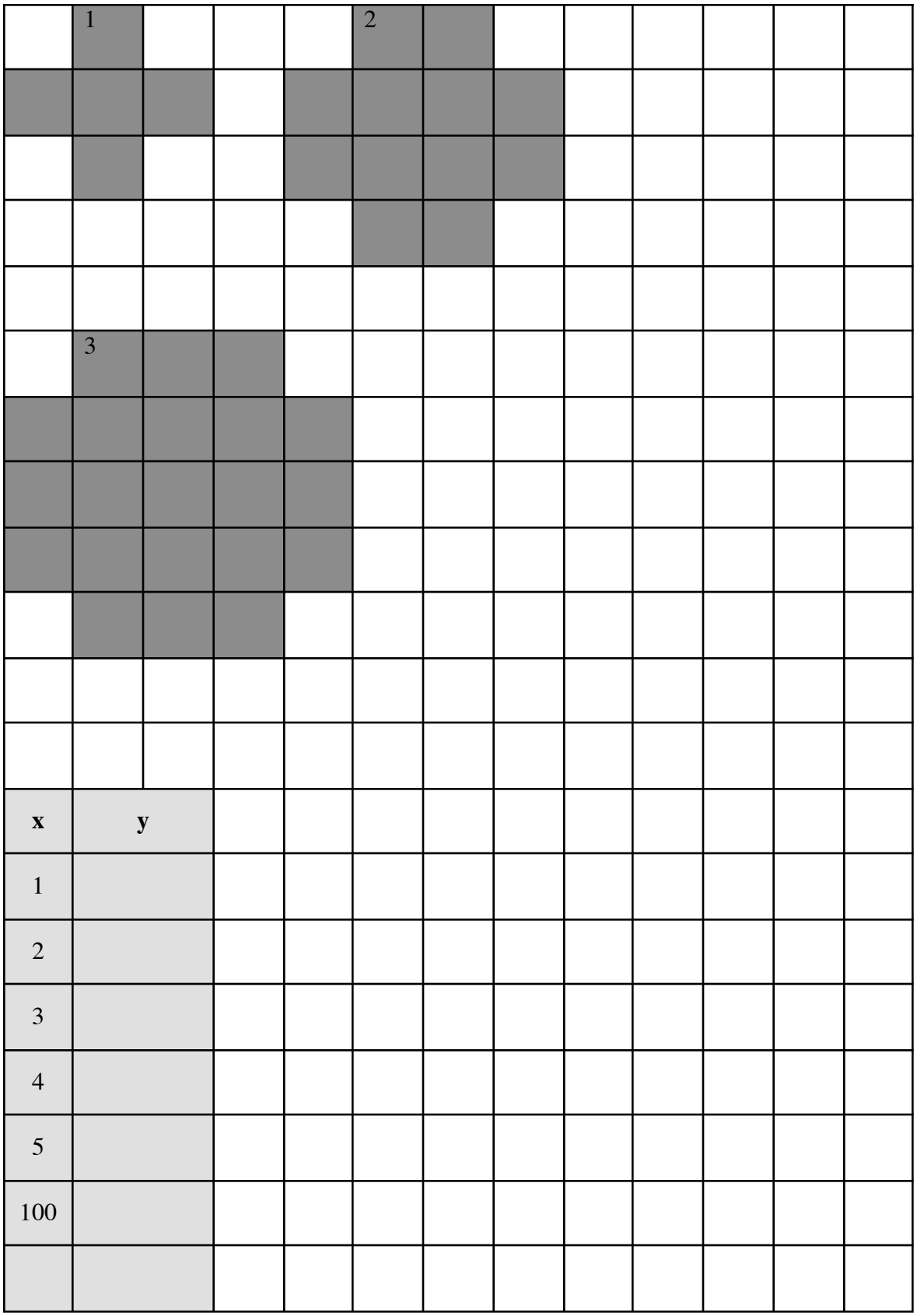

x	y
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From Boaler, 2008, p. 60

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x	**y**											
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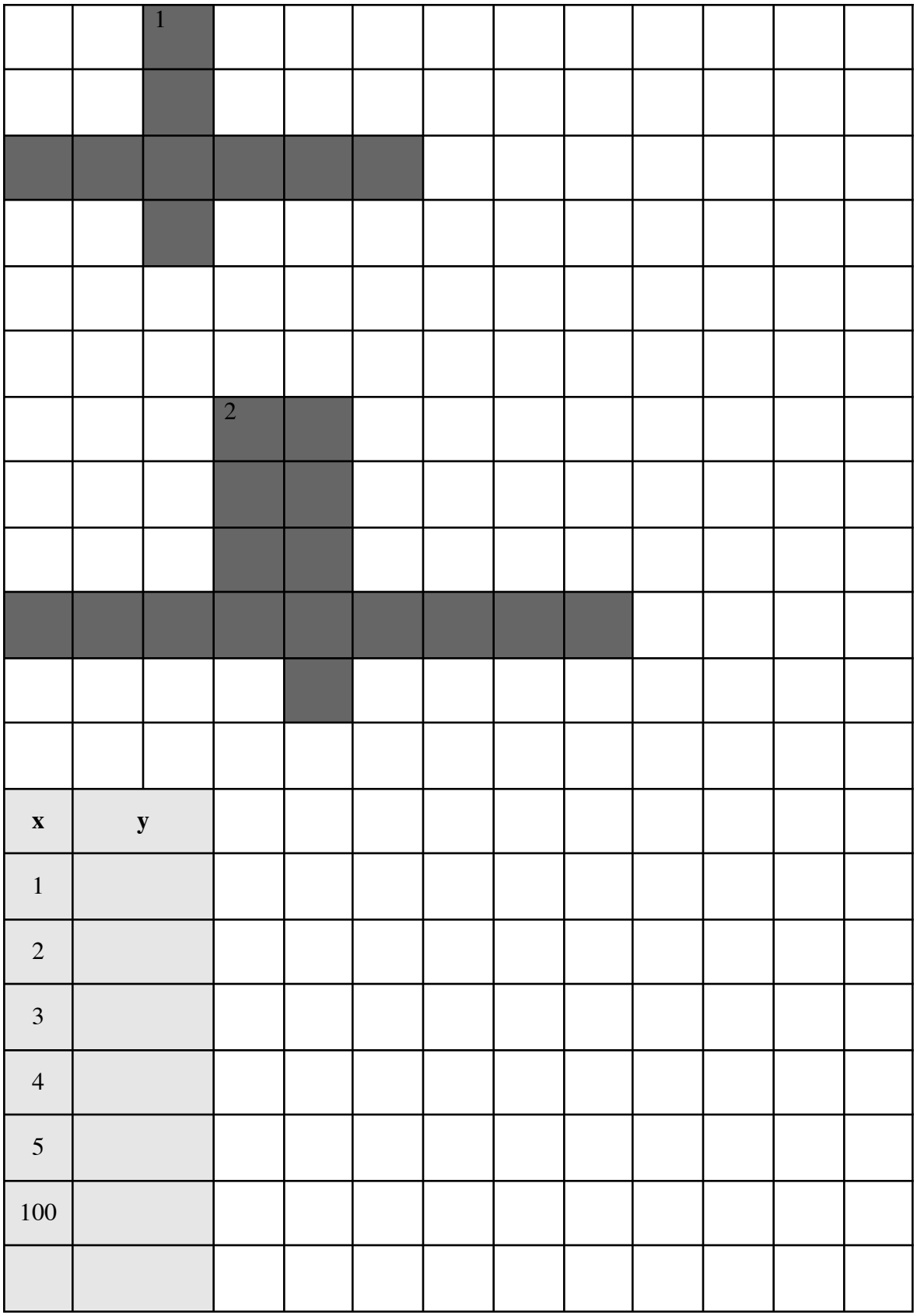
From Boaler, 2008, p. 64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From Boaler & Humphreys, 2005

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x	**y**											
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1		2					3					
x	**y**											
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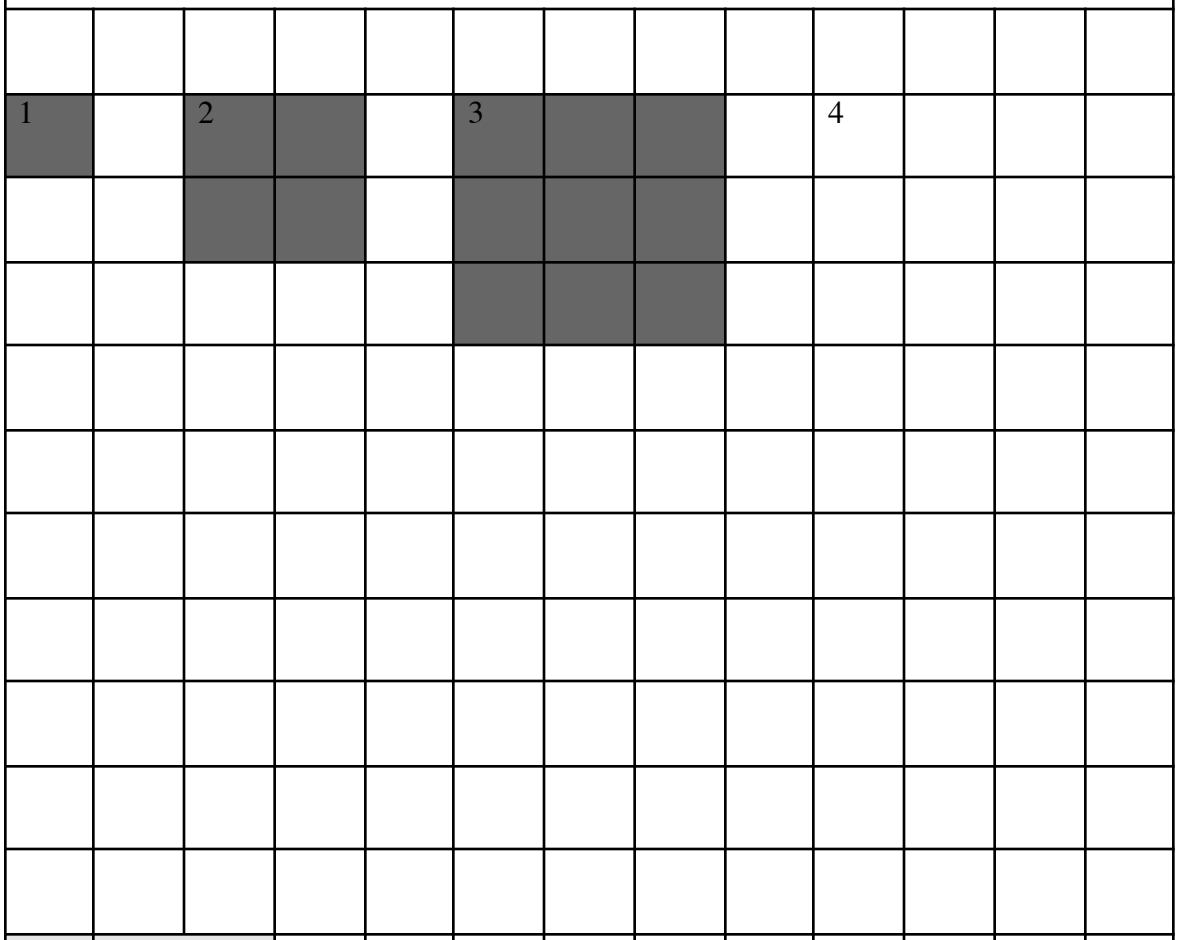
*Zack & Reid, 2003: 3 pyramids form an almost-cube: 1/3(n+1)(n+1/2)

For this one, count squares of all sizes – how many squares in the chessboards?



x	**y**
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How many ways can you arrange x^2+2x+1? What if you use partial squares?

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		3										
x	**y**											
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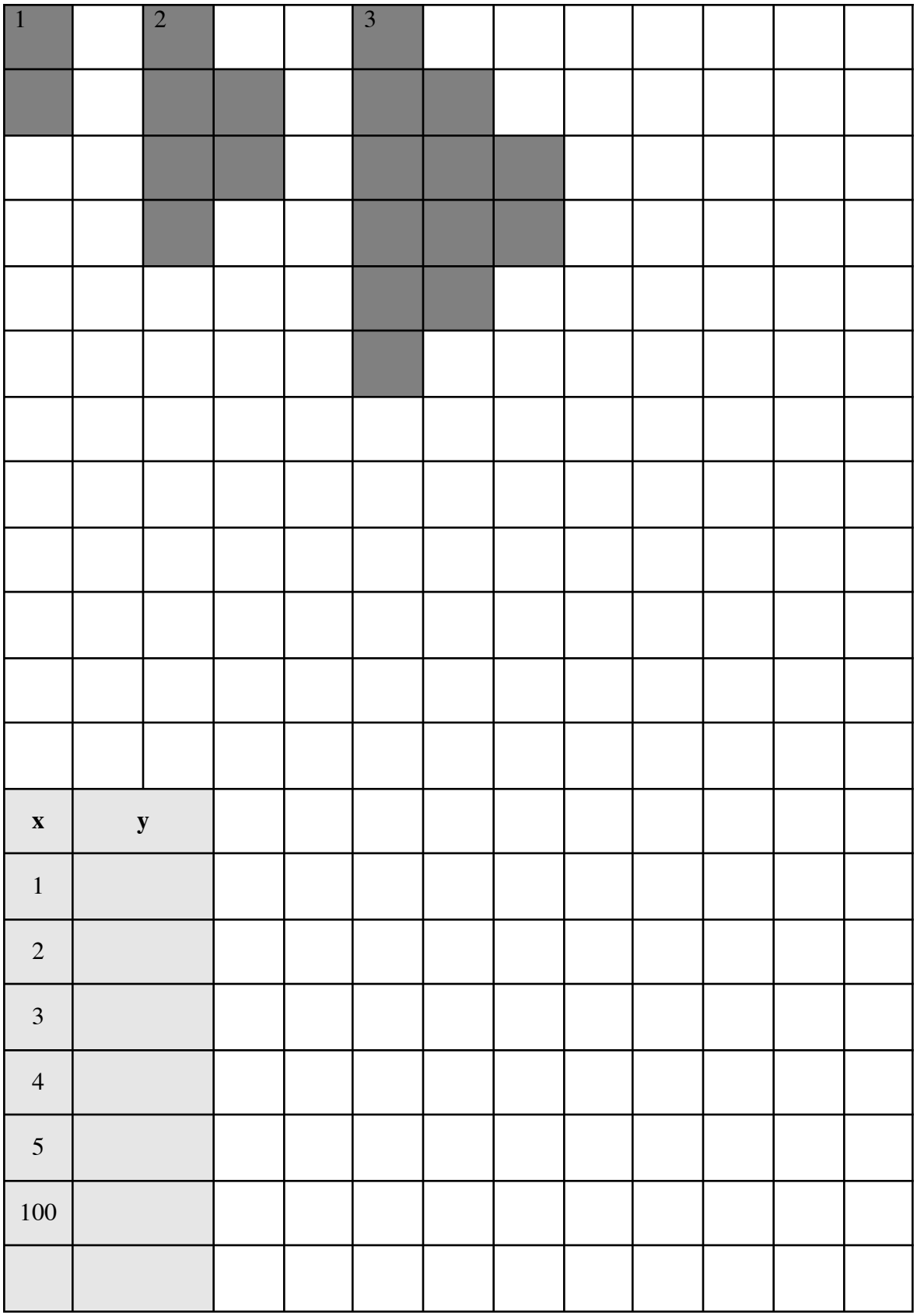
What would $4(x^2+2x+1)$ look like? x^2+2x+2? x^2-1? x^2-4?

3												
		3										
3												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x
y
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1		2			3							
x	**y**											
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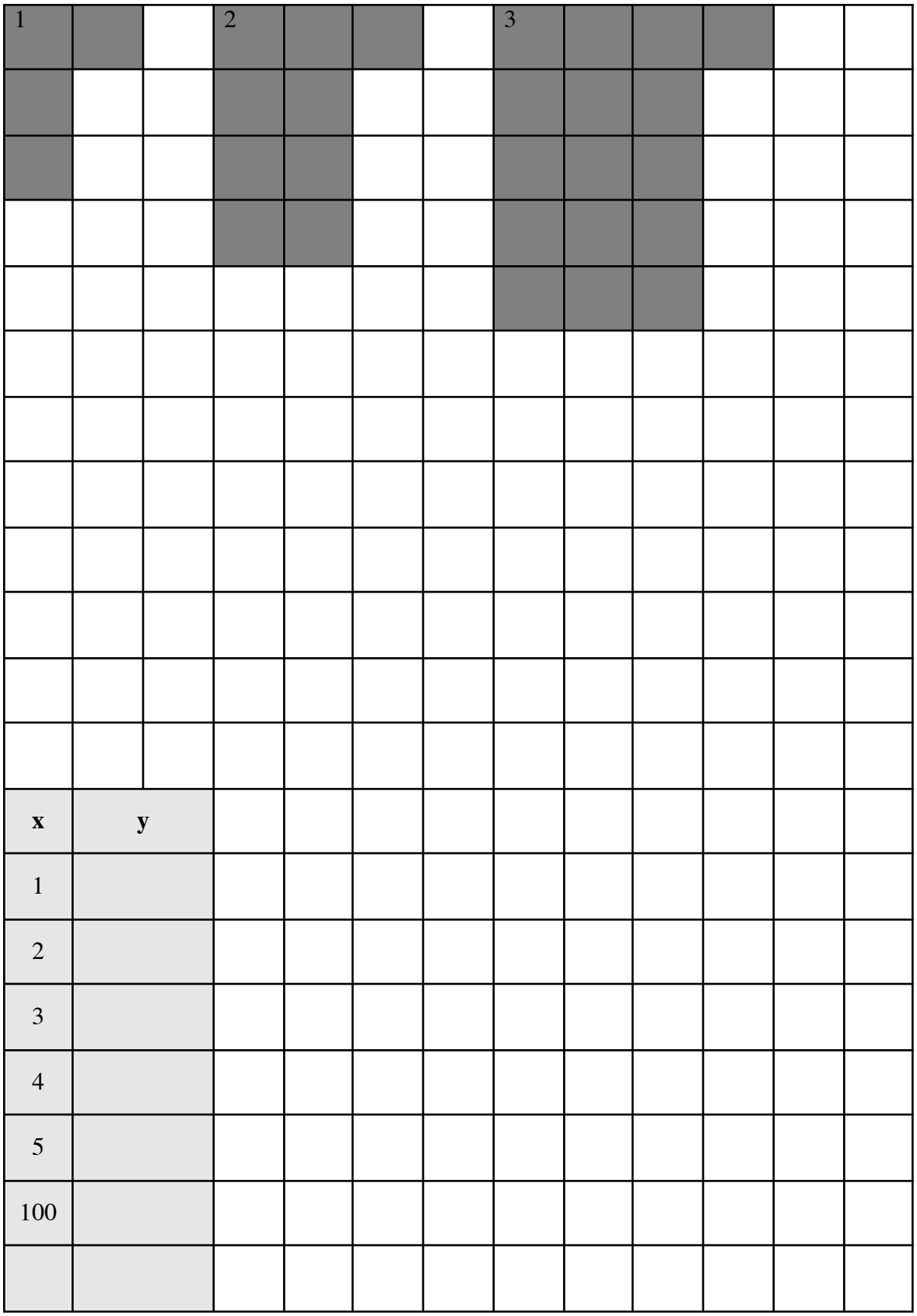

x	y
1	
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100	

1			2				3					
x	**y**											
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x	**y**											
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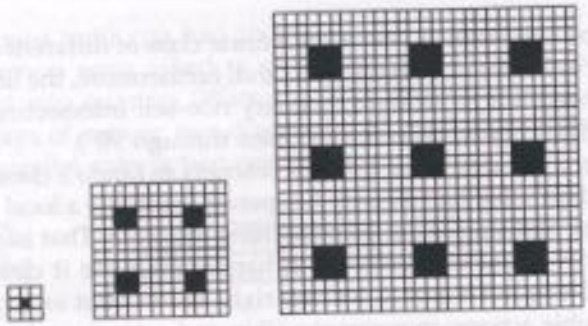
Task 38: High Degree

Write down a method of generating an infinite sequence of designs starting with the following three. Then find an expression for the number of white squares involved in the *n*th design.



Make up more spatial sequences that can be given in two dimensions and whose formulae involve degree three or four.

Watson, A. & Mason, J. (2005). *Mathematics as a constructive activity: Learners generating examples.* Mahwah, NJ: Erlbaum.