

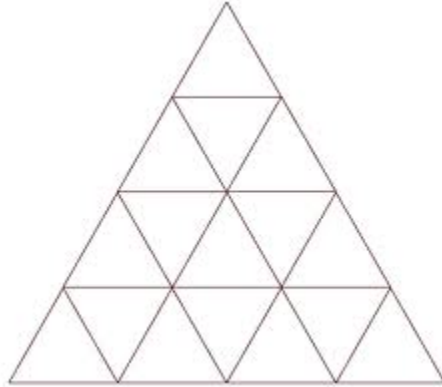
Problem Set

Alberta Institute – January 26, 2012

1. On a table there are 1001 pennies lined up in a row. I then come along and replace every second coin with a nickel. After this, I replace every third coin with a dime. Finally, I replace every fourth coin with a quarter. After all this, how much money is on the table?
2. A4 paper, the standard everywhere in the world but North America, has an interesting property – when you fold it in half it fits into an A5 envelope AND the ratio of its dimensions stays constant. If A0 has an area of 1 m^2 what are the dimensions of A4 paper?
3. David and Aaron each make a \$10 bet on a coin toss game. The game goes as follows: David is heads and Aaron is tails and the first to get five wins. After six coin tosses their math teacher interrupts the game. At that point David has two heads and Aaron has four tails. The teacher takes away the coin so the game cannot continue. Given the score, how should they split the \$20 pot?
4. In a game of chance a single player draws two blocks from a bag. If the two blocks are the same colour he wins. If they are a different colour then he loses. If there are only two different coloured blocks, how many blocks of each colour are needed in the bag to make the game fair? Find ALL the solutions.
5. On the way to work the other day I averaged 50 km/h. When I got there I realized that I it was Saturday. So, I hurried back to the house. How fast would I have had to drive home in order for the average speed of the total trip (to and from work) to be 100 km/h?
6. You have 10 silver coins. How much money might you have?
7. Three men, two orangutans, and a gorilla have to cross a stream. There is only one boat and it can only carry two of the six characters. Each of the men and the gorilla can paddle the boat by themselves (with or without a passenger). The problem is that if at ANY TIME there are more apes than humans in one place the apes will beat the humans to death. How do you do it?
8. A watermelon is 99% water. A 5 Kg watermelon is left in the sun until it is only 98% water. How much does the watermelon now weight?

9. At a new development a company is putting in streets and streetlights. The city bylaw states that there must always be a streetlight at every intersection. What is the maximum number of lights that would be necessary for n roads?
10. A wooden meter stick is laid on the floor with one end against the wall. The point 99 cm from the wall is marked and the ruler is then bowed until the end of the ruler meets this mark. How high is the bow?
11. When my son was very young he discovered that he was the same height sitting (on a 24 inch bar stool) as he was standing up. How tall was he?
12. On a unit circle the tangent of a standard position angle has always been discussed as the slope of the terminal arm. However, this does not mesh with our geometric understanding of tangent. But, there is a line which is tangent to the unit circle AND whose length is equal to the tangent of the angle. Find it. Also find the cotangent, secant, and cosecant lines.
13. From calculus we know that $\frac{\sin \theta}{\theta} = 1$. The formal proof of this is complex and involves the use of the squeeze theorem. However, an informal proof involves the recognition that the ratio in question is really just the ratio of a chord to its related arc. Find this.
14. We know that every odd number can be expressed as the difference of squares. Is the same true for even numbers?
15. How many ways to make a dollar using only silver coins?
16. The latest multi-level marketing scheme involves the sale of coffee. In this scheme new recruits are put into nodes of a perfect binary tree. It cost \$600 to sign up. Of this \$600 the person immediately above the new recruit gets \$150, the person above her gets \$75, etc. To stay in the program each member is put on a auto-shipment of coffee at a cost of \$75/month. Of this \$75 dollars, every person above the payee receives \$2. Where is the profit for the company? Where is the profit for members?

17. Consider the diagram below. How many upright triangles are there in this image? If the image grew until there were n rows, how many upright triangles would there be?



18. Find the ratio of the area of the larger triangle to that of the smaller triangle.

