

Open-Ended Math Problems – A Resource Packet

Created by Vladimir Stelkic

*This packet provides a sampling of open-ended problems that I used in my classroom. The hints are provided to help students who are struggling with problems initially and can be given one at a time, or from the beginning. The extensions are for early finishers or students ready to take the problem to the next level. **Also please refer to pages 8 and 9 for tips for teachers and tips for students!***

Jump to:

1. [The Weather Report Problem](#)
2. [The Locker Problem](#)
3. [The Flying V Problem](#)
4. [The Checkerboard Problem](#)
5. [Gauss Gone Mad Problem!](#)
6. [The “Terrible Twos” Problem](#)
7. [The “In-Between” Problem](#)
8. [Cookie Jar Problem](#)
9. [The Coins Problem](#)
10. [The Magic Bag Problem](#)
11. [“Crazy Fractions” Problem](#)
12. [Tips for Students](#)
13. [Tips for Teachers](#)
14. [References](#)



- 1. The weather report problem:** The weather is reported every 9 minutes on ABC and every 12 minutes on CBS. Both stations broadcast the weather at 1:30. When is the next time the stations will broadcast the weather at the same time?

Hints:

- Let's practice counting/working with time...
 - What time is 30 minutes after 1:15?
 - What time is 25 minutes before 4:15?
 - What time is 50 minutes after 3:11?
 - What time is 42 minutes before 6:17?

- Start by listing all of the times that ABC broadcasts the weather
- Now list all of the times that CBS broadcasts the weather. What time do they have in common?

Extensions:

- When was the last time (before 1:30) that the weather was reported at the same time?
- List the next 5 times that the weather will be reported at the same time. How often does this happen? Explain why this number is significant.
- Suppose the CBS changed to reporting the news only every 24 minutes. When is the next time they will report the news together?
- Since CBS has been getting low ratings on their weather program they have decided to only report the news every 45 minutes. After 1:30, when will the two stations report the weather together again?
- Are there any numbers that the stations could choose that would make it so they never report the weather at the same time? Explain.



2. The Locker Problem: Imagine High Tech Middle decided to install lockers for each of the 300 middle school students. The lockers are numbered from 1 to 300. When the High Tech Middle students return from summer vacation, they decide to celebrate the lockers by working off some energy.

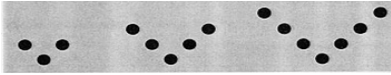
- The first student goes to all of the lockers and *opens every locker*
- The second student then goes along and *shuts every other locker*.
- The third student *changes the state of every third locker*. (if it's closed, they open it, if it is open, they close it)
- The fourth student *changes the state of every fourth locker*.
- The fifth student changes the state of *every fifth locker*, the sixth *every sixth locker*, and the pattern continues until all students have marched.
- After all the students have marched, which lockers remain open, and which ones are closed?

Hints:

- Let's start by looking at the first ten lockers.
 - Can you draw a picture of these ten lockers?
 - Show what happens to each of the lockers after the first student marches (make a symbol like "o" for open).
 - Now show what happens after the second student has marched. Which are open now?
 - Continue this pattern until all ten students have marched. You should have three lockers open... what are they?
 - Now extend this to include the first 20 lockers and see which lockers are open
- What are the 'perfect square' numbers?
 - Can you list their factors? What might this have to do with the locker problem?

Extensions:

- What if the students were not in order when they went marching to open and close lockers. Would the same lockers be open in the end? Why or why not?
- Which lockers were only touched by two students? Were any lockers touched by only three students? What is significant about these numbers?
- Which number locker was touched by the most students? What is significant about this locker?



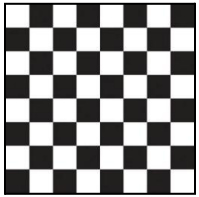
3. **The Flying V Problem:** Groups of ducks often fly in a v pattern as seen below. The first three possible flying v patterns that birds could fly in are seen here. How many birds will be in the 20th v-pattern?

Hints:

- Draw the next 5 v-patterns. How many birds are in each?
- Make a table displaying what you see so far.
- Do you see any patterns in the number of birds so far? If not, keep drawing pictures and extending your table.

Extensions:

- How many birds will be in the 100th v-pattern? Justify your answer using pictures, tables, explanations, etc.
- Which v-pattern will have 999 birds in it? Explain.
- Write an algebraic expression that represents the number of birds in each v-pattern.
- Why do birds fly in v-patterns? Research this topic and be prepared to share with the class later!



- 4. The Checkerboard Problem:** Macy and Nick were playing checkers when Macy suggested, "Let's count all the squares on the checkerboard!" Nick said, "That's easy, all we have to do is multiply 8 X 8." "Oh no, Nick. There are many more... and lots of different sizes," Macy responded.

Help Macy and Nick count all the squares on the checkerboard.

Hints:

- How many boxes are there that are 1x1? 2x2? 3x3? Make sure you aren't missing any!
- Can you make a table to display the number of boxes that are each size? Look for patterns!

Extensions:

- What if the checkerboard were 9x9? How many boxes would there be in this case?
- Why are "perfect square" numbers popping up again in this problem? What do they have to do with checkerboard?

I think I'm going mad!



- 5. Gauss Gone Mad Problem!:** We all remember our friend CARL FRIEDRICH GAUSS (he was the famous mathematician who quickly added the numbers from 1 to 100, amazing all of his teachers!!!). Today, Gauss has an extra challenge for you. He wants you to add the numbers from 1 to 100 again, but this time with alternating signs. Here is the beginning of your list of numbers:

1, -2, 3, -4, 5, -6, 7, -8, 9, -10, 11, -12, 13, -14, 15, -16, ... , 99, -100

What is the sum of all of these numbers? Use your knowledge as an integer wizard to look for shortcuts, patterns, and the sum!

Hints:

- Try breaking the numbers from 1 to 100 into pieces.
- What is the sum of 1 and -2? What is the sum of 3 and -4? What is the sum of 5 and -6? Could this help you find the sum of all numbers?

Extensions:

- Does it matter that the odds are negative and the evens are positive? Check the sum of these numbers:

-1, 2, -3, 4, -5, 6, -7, 8, -9, 10, -11, 12, -13, 14, -15, 16, ... , -99, 100

- Suppose Gauss asked you to find the sum of all of the even numbers with alternating signs? (2, -4, 6, -8, 10, -12, 14, -16, 18, ..., -98, 100) Would the sum again be -50 just like in the first problem? Why or why not?
- Check the sum of the sequences with multiples of three, four, and five. Do you notice any patterns?



6. The “Terrible Twos” Problem:

“My favorite number is TWO!” shouted Tyler.

“Why?” asked Trevor, “There’s nothing special about it.”

“That’s not true. You can make any number in the whole world, using just twos... and a few operations, of course,” Tyler argued back.

“I don’t believe you... prove it!” responded Trevor.

With his head held high, Tyler began, “Ok... well $1=2\div 2$ AND $2=2+(2-2)$ AND $3=2+(2\div 2)$ AND ...”

Do you think that Tyler is correct? Can you generate EVERY integer from just TWOS and the four arithmetic operations?

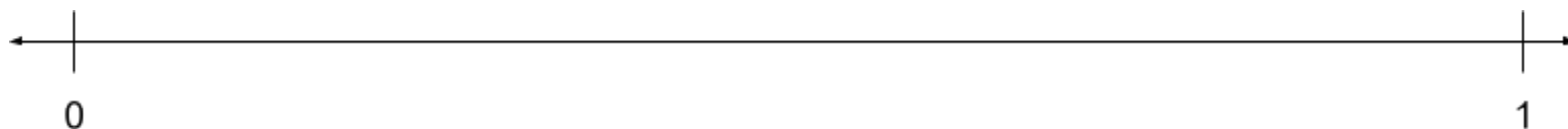
Hints:

- The answers for 1, 2, and 3 are given to you. Try 4 (this should be easy). Now try 5. How can we make an odd number when working with only even numbers? (Look at how I made 3 for an idea!)
- Try organizing your answers into a neat chart to help you notice common patterns.

Extensions:

- Try to find the simplest/shortest way to make the numbers 1-100 using only TWOS.
- Suppose you were given the numbers 2, 4, and 6. Could you make every single positive integer using these three numbers and the four arithmetic operations?

- 7. The “In-Between” Problem:** On the number line below, label as many possible numbers that you can between the integers 0 and 1 (*please do not use decimals for this activity, but DO put them in the correct location*):



Hints:

- What number is right in the middle of 0 and 1? Place this on your number line
- What number is right in between 0 and $\frac{1}{2}$? $\frac{1}{2}$ and 1? Place these on your number line.

Extensions:

- Does your number line show fractions like $\frac{1}{7}$? $\frac{3}{11}$? Why or why not? Place at least 10 other “more obscure” fractions like this on your number line.
- Make a beautiful number line for us to hang in the room that has at least 30 fractions in the correct locations. This will be for us to refer to throughout the rest of the year!



- 8. Cookie Jar Problem:** There was a jar of cookies on the table. Daniel was hungry because he hadn't had breakfast, so he ate half the cookies. Then Tannia came along and noticed the cookies. She thought they looked good, so she ate a third of what was left in the jar. Lisette came by and decided to take a fourth of the remaining cookies with her to her next class. Then Hannah came dashing up and took a cookie to munch on. When Avery looked at the cookie jar, he saw that there were two cookies left. "How many cookies were there in the jar to begin with?" he asked.

Hints:

- Try working on the problem backwards. Avery found two cookies. How many cookies were there when Hannah got there?
- If Lisette took $\frac{1}{4}$ of the cookies and there were 3 left, how many did she take? Draw a pie chart to help you answer this.

Extensions:

- Visually display your results using a pie chart, rectangle, or table to show that you completely understand this problem.
- The answer to this problem, 12, is the LCM of the denominators of the fractions ($\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$). Is this a coincidence? Write another problem that is similar to this and see if this trend continues.



9. The Coins Problem: I have *twenty four coins* in my pocket. I count up the total and it adds up to \$3.78... enough to buy an awesome lunch! What fraction of the coins in my pocket are quarters? What fraction of the coins are dimes? Nickels? Pennies?

Hints:

- Try starting with quarters. How many quarters could fit into \$3.78? What else would you need? Is this 24 coins?
- Continue guessing and checking in the method above.

Extensions:

- How many answers are there to this problem? Find as many as you can!
- Could there be an algebraic method of figuring out multiple solutions? Try making variables for each of the coins and seeing what you come up with!



10. The Magic Bag Problem: A fortune teller has a magic bag with the numbers (1, 2, 3, 4). Two numbers are chosen at random from the bag by a man in the audience. The two chosen numbers are used as numerator and denominator of a fraction. The fortune teller says that if that fraction that is made is equivalent to a whole number, the person will have *a good life forever*. If what is drawn is an improper fraction, the person will have *something bad happen* to them very soon. And if the fraction is proper, then the person will *grow wings and fly away*.

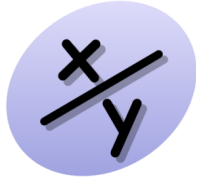
- What fraction of the possibilities will produce *a good life forever*?
- What fraction of the possibilities will cause *something bad to happen*?
- What fraction of the possibilities cause *wings to grow and fly away*?

Hints:

- Define improper, proper, and whole number equivalents for the students.
- Try making slips of paper with 1, 2, 3, and 4 on them and start drawing them two at a time. Make a list of all of the fractions you could get by doing this.

Extensions:

- Would this man's odds stay the same if he were given the numbers (1, 2, 3, 4, 5)?
- What about (1, 2, 3, 4, 5, 6)? Is there a pattern here?



11. “Crazy Fractions” Problem:

$$\frac{11}{aa} - \frac{11}{bb} = \frac{11}{cc}$$

What could a , b , and c be to make this a true equation?

Hints:

- Don't forget that once you plug in a number for a , b , and c , you will have to give them all a common denominator to see if it works.
- Try $a=2$, $b=3$, and $c=4$. Set up the equation and then check it using common denominator. Does it work? Now find some more!

Extensions:

- Can you find any other combination of numbers that work? Look for the pattern.
- (a real 11th grade problem!) Which of the fractions of the form QUOTE can be written as the sum of two distinct unit fractions (A unit fraction is of the form $1/n$).

Tips for Students
... written by students!

- Don't get frustrated... it's ok to solve the problem in different ways from your friends.
- Don't just read it and think "Oh, this is too hard" and then goof around. Once you actually think about the problem, then you'll get it.
- Don't stress out, just remember what math is: patterns!
- A good way to start is to draw out your thoughts.
- Always have a reason for the answer you got.
- These problems can seem endless but once you get them, you always remember them.
- Don't worry about doing this problem quickly or in one piece. Break it down into multiple ways and solve it step by step.
- Don't worry about anyone who is finished before you.
- Try every pattern you can think of and don't be afraid to tell your group because your idea might be the formula to solving the problem!

Tips for Teachers

Some strategies for implementing open-ended problems in your classroom

- ***Solve the problem first yourself.*** It is important, before you give a problem to students, to be aware of the basics of the problem so that you can help struggling students gain a solid foundation. Similarly, it is good to know some related extension exercises to deepen students' understanding.
- ***Encourage group work and conversation.*** Because open-ended problems allow multiple approaches, they offer an ideal context for students to learn from each other and share responsibility for finding solutions. Allow freedom of movement. When we work on these problems, I allow my students to go anywhere in my room or our common spaces to work. Sometimes a change of scenery is what they need to jump-start their thinking.
- ***Refrain from giving hints for a set period of time.*** Some students are apt to give up the moment they feel confused. By holding off on giving hints, you are encouraging them to “think for themselves,” and this is where immense growth happens. After a set time, offer support by encouraging them to draw and label a picture of the scenario. After this, you could pair them with a partner or group who seems to have a good start on the problem and who can offer some advice.
- ***Hold off on telling them if their answer is “right” or “wrong.”*** If students say that they are done, I like to ask them how much they would be willing to wager on their answer. This gives me an idea of how confident they are in their thinking and encourages “guessers” to keep working at it. Another question I have posed with students is “could you solve this another way?” which often sets them off to work again trying to find other methods to “check” their solution.
- ***Have students post and explain work on the board.*** Articulating their process is often the hardest part for early finishers. Encourage them to work out their solutions on the board in such a way that younger students would understand their thinking. Sometimes even incorrect approaches are helpful to put on the board so students can see where their thinking went wrong.

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