Proportional Reasoning

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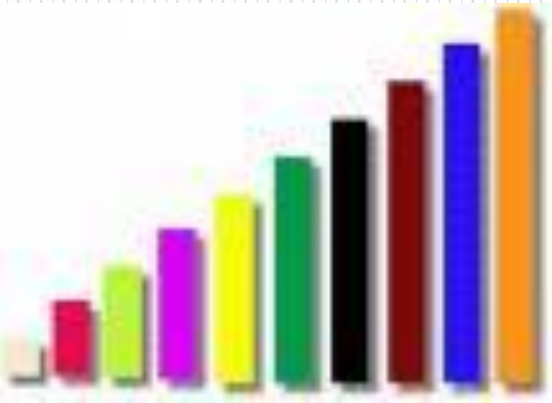
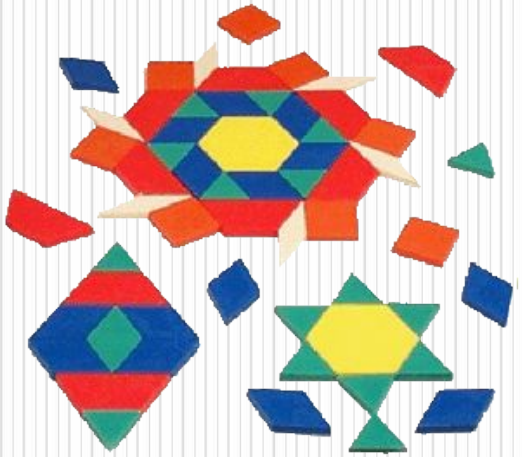
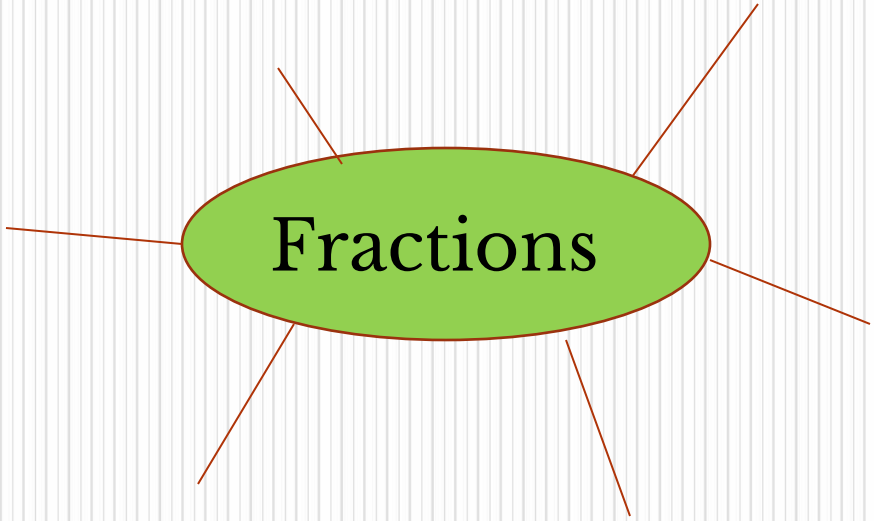




On a large sheet, write down everything you know about:



One of the areas most frustrating for teachers and students alike is the study of fractions.

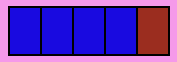
The main reason students have difficulties with fractions is that they seem to want to memorize formulas or algorithms instead of understanding them.

Manipulatives, when used to introduce concepts about fractions, help students understand the ideas about fractions. Pattern bloc and fraction blocks have many uses in learning mathematical concepts, but they are especially useful in learning about fractior

Did you include...

Area or Region

4/5 of this region is blue



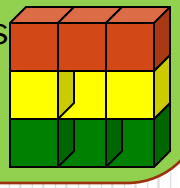
The ratio of red hexagons to total hexagons is 2 : 5

or

2/5 of this set of hexagons is red

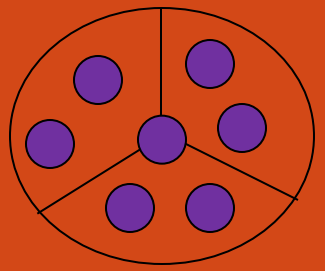
Measure

1/3 of the volume of this cube is made up of green cubes



Division

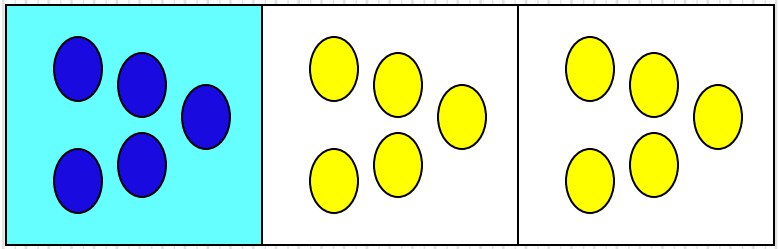
If you divide 7 counters among the 3 equal parts of this circle, there are 2 1/3 counters in each part. 7 ÷ 3 = 2 1/3



Set or group

2/5 of the group of counters are red

How is a fraction of a set like a fraction of a region?



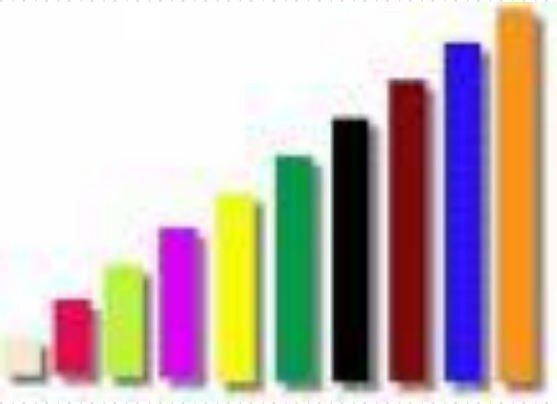
1/3 of the rectangle is blue

1/3 of 15 counters is 5.

"Making Math Meaningful to Canadian Students" Marian Small, Nelson

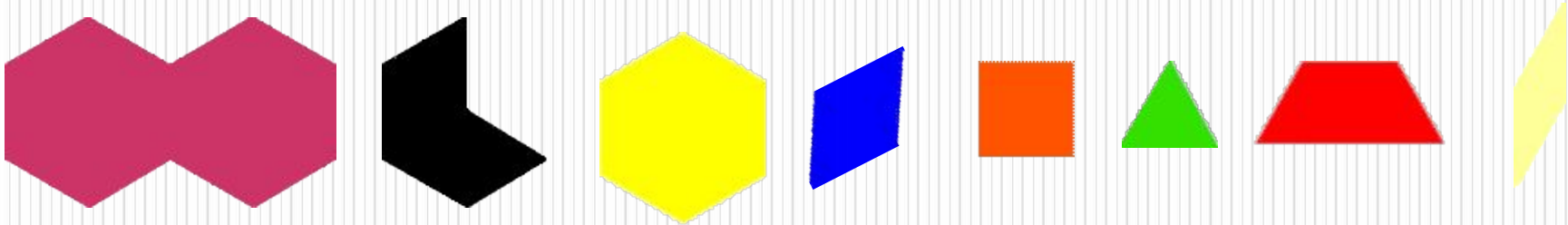
Exploring Fractions with …

Fraction Blocks
Cuisenaire Rods



Getting to know the fraction blocks

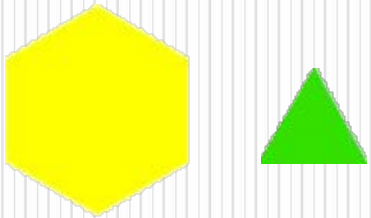
This lesson allows the students to become familiar with the blocks and their relationships.



Fraction blocks

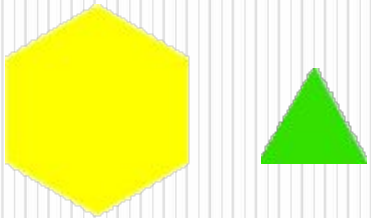
Pattern blocks

Fractions – *What's My Value?*



Find the value of the green triangle given that the value of the yellow hexagon is 1.

Fractions – *What's My Value?*



Find the value of the green triangle given that the value of the yellow hexagon is 1.

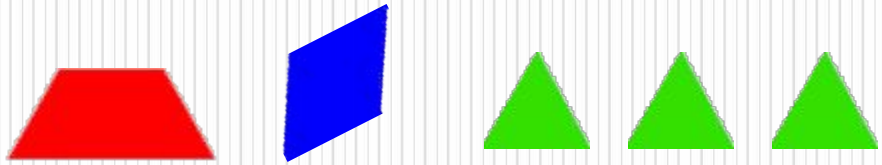
Now figure out the values of a blue parallelogram and

a red trapezoid when the value of the yellow hexagon is 1.

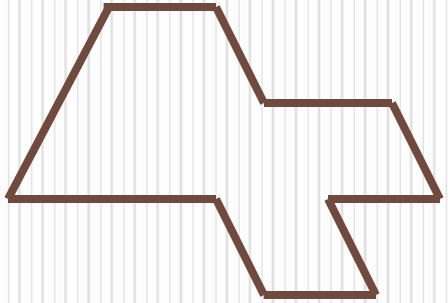
Fractions – *Pattern Block Riddles*

- I have hidden five Pattern Blocks.
- The three smallest blocks exactly cover the largest block.
- One of the blocks covers two-thirds of the largest block.

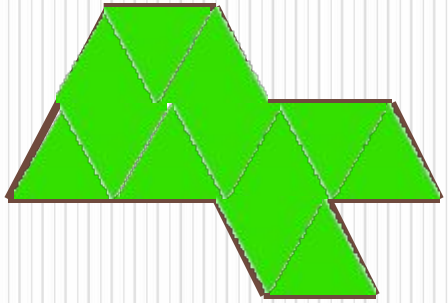
Click to reveal a possible answer



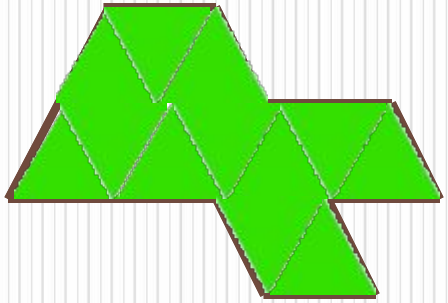
If the  = 1, what is the value of the following shape



If the  = 1, what is the value of the following shape



If the = 1, what is the value of the following shape



If the = ½ , what is the value of the following shape

If the = ¼ , what is the value of the following shape

Why does the same shape keep switching values? Explain your reasoning.

How can you find the value of a large Pattern Block design when you know the value of 1 block?

Working with a partner, create a Pattern Block design using 20-30 blocks of at least 3 different colors. Use only red, green, yellow, and blue blocks.

- Draw your design on triangle paper and color it to match the blocks you used.
- Figure out the value of your design if the hexagon equals 1. Write this number on the back of your paper
- Now figure out the value of your design if the trapezoid equals 1.
- Finally, figure out the value of your design if the blue parallelogram equals 1.
- Record each of these values on the back of your design.
- Exchange designs with another pair and try to figure out the values of their design in each of the three situations (hexagon = 1, trapezoid = 1, and blue parallelogram= 1).

If your results are different, work out the values together.

Fractions – *What's My Value?*

Questions to ask while the children are engaged.

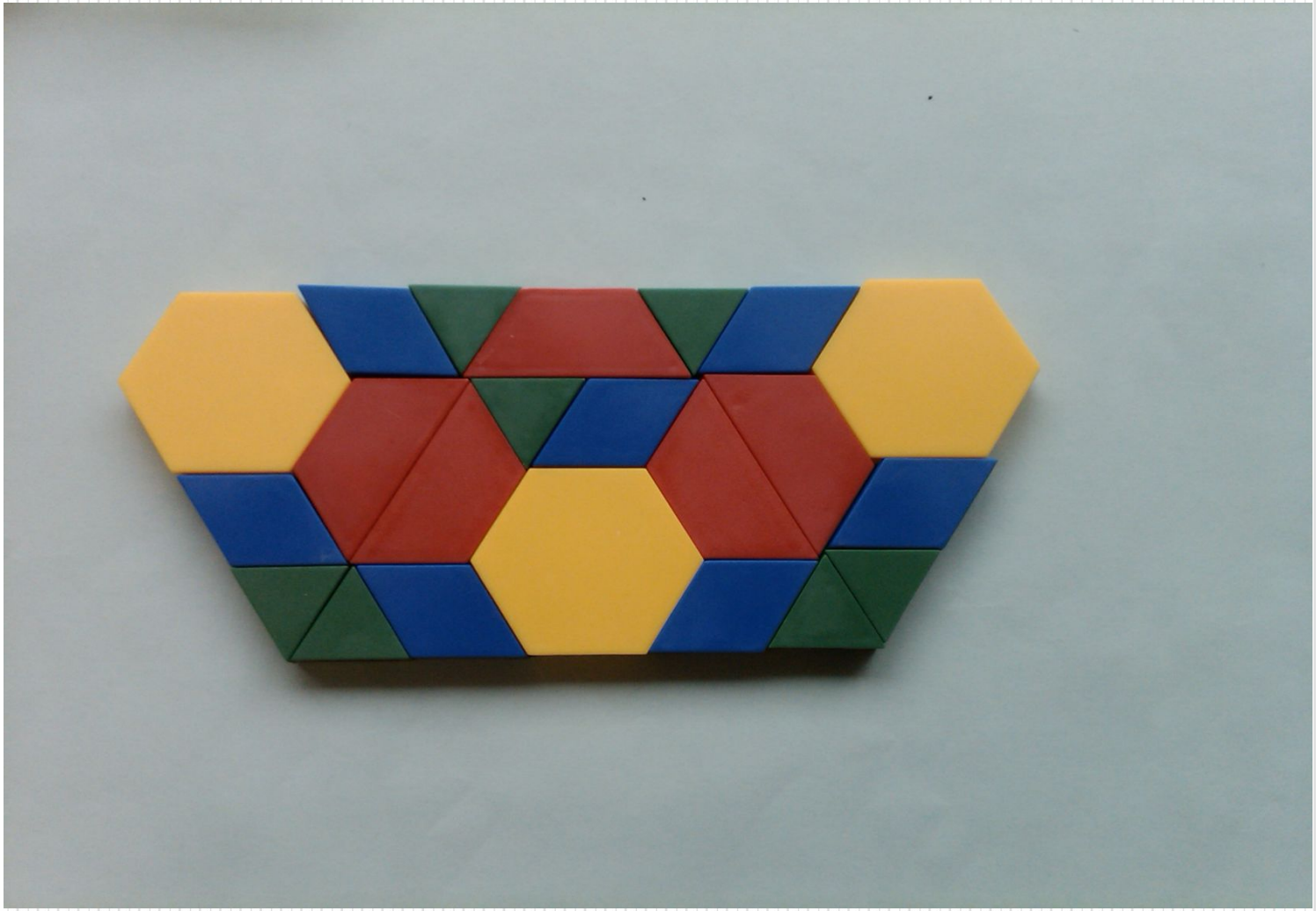
How did you figure out the value of the designs when the hexagon was '1'? When the trapezoid was '1'? When the parallelogram was '1'?

How did you determine the fractional parts?

Did you notice any patterns in the different values for your designs? If so, what?

How can you explain the patterns in the values?

If a hexagon = 1 Total = 9
If a trapezoid = 1 Total = 18
If a parallelogram = 1 Total = 27



Exploring Fractions with Pattern Blocks

1. Build a triangle that is 1/3 green and 2/3 red.
2. Build a triangle that is 2/3 red, 1/9 green and 2/9 blue.
3. Build a parallelogram that is ¾ blue, and ¼ green.
4. Build a parallelogram that is 2/3 blue and 1/3 green.
5. Build a trapezoid that is ½ red and ½ blue.

"MATHEMATICS: with manipulatives", Marilyn Burns

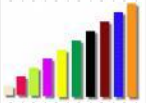
First to Finish – using cuisenaire rods

If the dark green rod equals ‘1’, what is the value of the following rods? Defend your position using the rods and what you know about fractional relationships

Light green = _____

Red = _______

White = ______



If the dark green rod equals ‘1’, what is the value of the following rods? Defend your position using the rods and what you know about fractional relationships

Light green = 1/2

Red = 1/3

White = 1/6

Dark Green					
Green		Green			
Red		Red		Red	
White	White	White	White	White	White

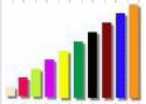
Make trains of these rods and place them in front of you to help support your positions

Create new trains as your value of the whole changes in order to answer the following questions:

Orange = 1

Brown= 1

Orange + Red = 1



Good questions to ask:

- How many different designs can you make that are ¾ red and ¼ yellow?
- The answer is 3/7. What might the question be?
- One third of a class orders lunches from the cafeteria each day? How many students might be in the class and how many order lunches each day?
- How many different ways can you show 2/3?
- If you have the numbers 1,2,3,4 how many non-equivalent fractions can you make?

**If you picked another set of number's would there also be 11 sets?

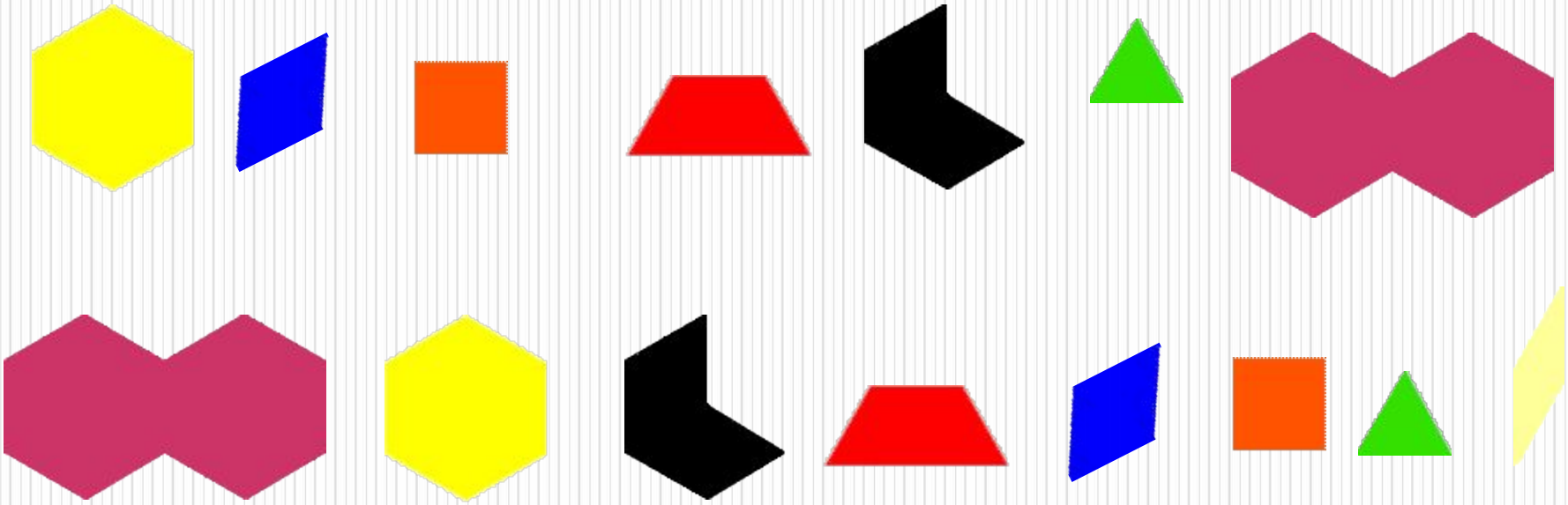
- Which is bigger

$\frac{201}{301}$ or $\frac{2}{3}$?

Good Questions for Math Teaching: Why Ask Them and What to Ask by Sullivan and Lilburn

Extension
Fractions: Comparing Area

Put the pattern blocks in order from least to greatest area and explain why you arranged them as they did.



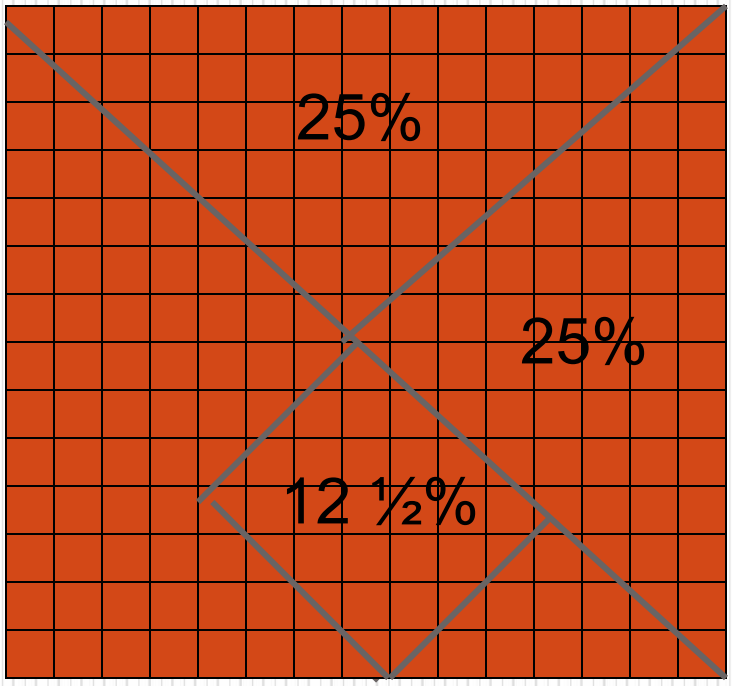
Square Cover-Up

Percents, Estimation, Area

How can you find the percent of an entire area that shapes cover?

- Work in a group. Decide who will be the Percent Maker. Let everyone else be Percent Finders. The Percent Maker uses from 3 – 6 pieces to make a design that covers part of a 100 square grid.
- The percent finders copy the design on their grids and discover the percentage of area that is covered.

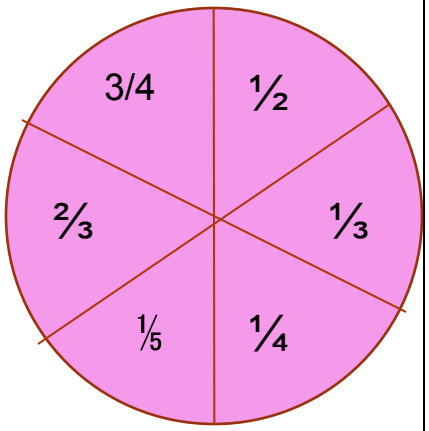
Extension – Use square paper without grid marks.


25%
25%
12 ½%

Fraction Fracas

In this game, students take turns finding pairs of Cuisenaire rods that represent a particular fraction in an effort to collect the most rods.

- This is a game for 2 – 4 players. The object is to collect the most rods.
- Players put all the rods in the center of the playing area.
- Each player, in turn, spins a spinner
- The player takes **two** rods that represent the fraction spun.
- Players lose a turn if no fraction may be created and the game is over when no one can make a fraction.



Exploring Decimals with …

Base 10 blocks

Decimals

If I use:

- a flat to represent one whole.
- a long to represent a tenth and
- a unit to represent hundredths, what numbers can I represent using exactly 4 pieces?7 pieces?10 pieces?

- Extend: What is the smallest and largest possible number?
- Extend: Place these numbers from largest to smallest.

A flat is 1 = (one whole)
A long is 0.1 = (one tenth)
A unit is 0.01= (one hundredth)

Decimals Mirrors

- Work with a partner
- Player #1 secretly models a decimal amount on a place value chart using flats, longs and ones.
- This partner records the value of the block collection and states the value aloud.
- The other partner ‘mirrors’ the amount on his place value chart.
- Now partners take turns naming amounts to add or subtract from their collections.
- After each partner has had three turns naming amounts to add or subtract, partners display their mats and compare their collections.

Good questions to ask:

- I am thinking of some decimal numbers between 1 and 2. What might they be? Give at least 15 answers
- Using only these keys on your calculator (‘5’, ‘.’, ‘4’ ‘+’, ‘=‘), what numbers can you make the calculator
- Represent 1.4 in a variety of ways.
- I added three numbers together to make exactly 4. What might the three numbers be?
- How many different ways can you make your calculator show 12.34 without pressing the decimal point button?

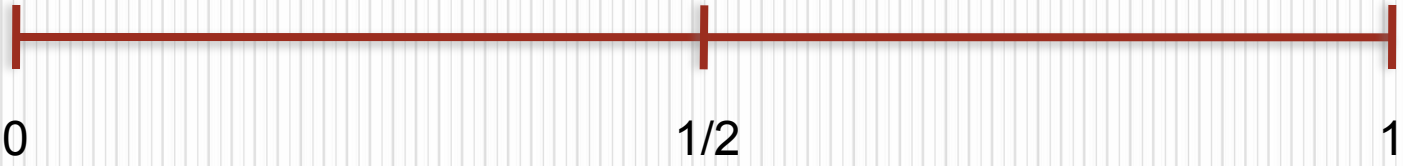
** may add ¼ and ¾ to benchmarks**

Decide whether each decimal is closer to zero, one half or one.

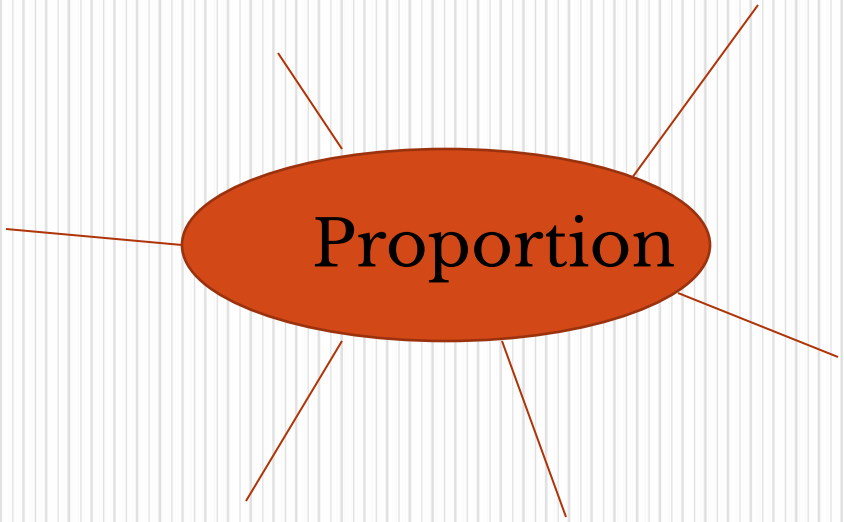
0.29
0.55
0.03
0.4
0.09
0.90
0.6
0.75

Benchmark decimals and fractions can serve as useful referent points. Being able to identify the relative value of decimals can help students make better comparisons between decimals and fractions.

Asking the follow-up question, "How do you know?" once an answer is given allows the teacher to hear students' understandings and misconceptions.



In 3 minutes write everything you know about proportion…. on the back of your ‘fraction’ sheet



3 Types of Proportional Reasoning

Missing Value Problems	Numerical Comparison Problems	Qualitative Comparison Problems
If 3 balloons cost $2.00, then how much do 24 balloons cost?	Which is the better value? - 3 balloons for $2 - 24 balloons for $12	What happens to the price of a balloon if you get more balloons for the same amount of money?

"Adding It Up" Helping Children Learn Mathematics, National Research Council

Who will complete the 10 km race first?

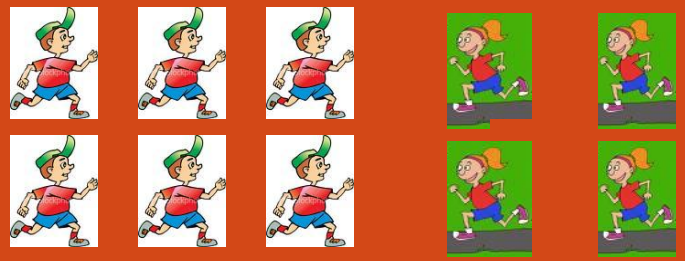
A person who runs 7 km/minute

A person who runs 8 km/minute

Students need to look at the relationship between the numbers instead of looking at one number in isolation.

"Making Math Meaningful to Canadian Students" Marian Small, Nelson

Exploring Ratios



Part to Part

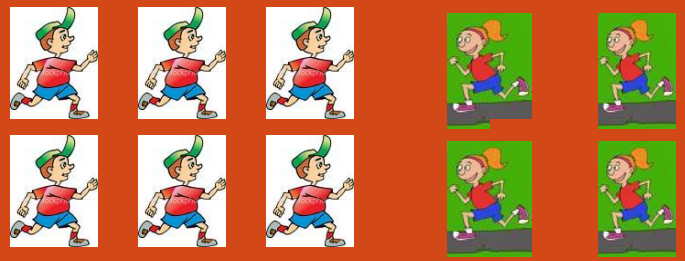
- _____ describes boys : girls
- _____ describes girls : boys

Part to Whole

- _____ describes boys : runners
- _____ describes runners : boys
- _____ describes girls : runners
- _____ described runners to girls

"Making Math Meaningful to Canadian Students" Marian Small, Nelson

Exploring Ratios



Part to Part

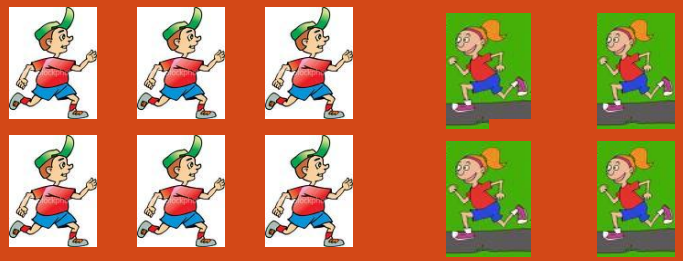
- 3 : 2 describes boys : girls
- 2 : 3 describes girls : boys

Part to Whole

- 3 : 5 describes boys : runners
- 5 : 3 describes runners : boys
- 2 : 5 describes girls : runners
- 5 : 2 described runners to girls

Any ratio can also be described as a fraction.
3:2 says that there are 3/2 as many boys as girls.

"Making Math Meaningful to Canadian Students" Marian Small, Nelson



Two ratios are equivalent if they represent the same relationship

If there are ….boys	There are …. girls
3	2
6	4
9	6
12	8
15	10

"Making Math Meaningful to Canadian Students" Marian Small, Nelson

4 Different Types of Ratio Problems

Choose one of the four problems below and be prepared to share how you arrived at an answer.

- Well-Chunked Measures

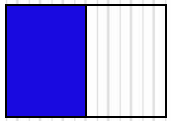
After 2, 5, and 7 hours of driving distances travelled were 260 km, 650 km, and 890 km. Did the travellers drive at the same speed?

- Associated Sets

7 girls are sharing 3 vegetarian pizzas. 3 boys are sharing 1 pepperoni pizza. Who gets more pizza, a boy or a girl?

- Part-Part Whole

Which shape is more blue?



- Stretchers and Shrinkers

Is an 8 x 10 enlargement of a 5 x 7 picture exactly the same as the original

"Making Math Meaningful to Canadian Students" Marian Small, Nelson

Exploring Ratios with *Cuisenaire Rods*

*Note: These rods are not to scale.

white = 1 cm.

red = 2 cm.

light green = 3 cm.

purple = 4 cm.

yellow = 5 cm.

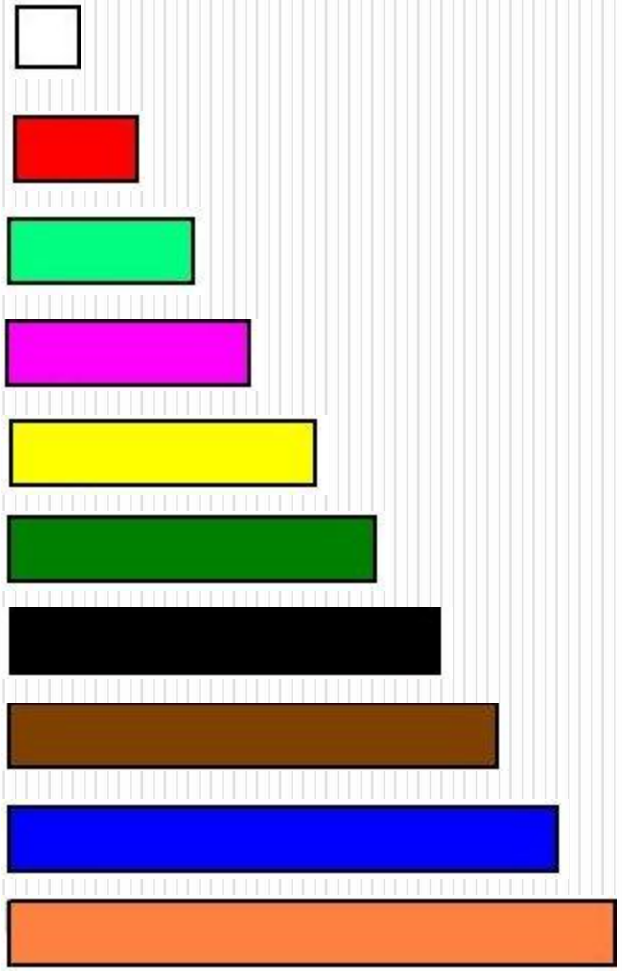
dark green = 6 cm.

black = 7 cm.

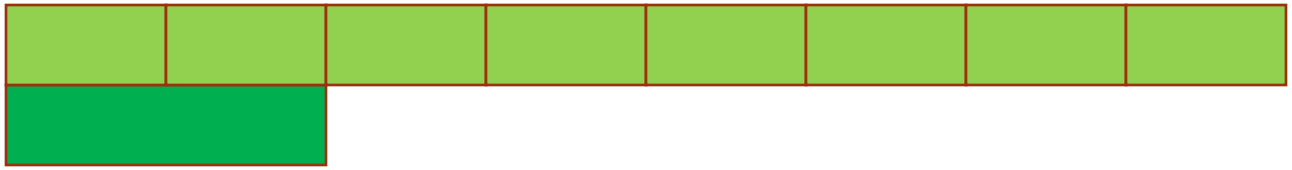
brown = 8 cm.

blue =9 cm.

orange = 10 cm.



Ratios



- Add light green rods to cover the complete width of the desk.

  How can you use this number of light green rods to find the width of the desk in dark green rods?
- Share your thinking.
- Confirm that half as many dark green rods are needed because two light green rods have the same length as one dark green rod.

If a desk is as wide as 22 light green Cuisenaire rods, how many of each of the other colours of rods would fit across the same desk.

- Work with a partner. Pick any other rod other than light or dark green and decide how many of that colour rod would fit across the desk if the rods were placed end-to-end like this:

- Record your estimate and the reason for your estimate. Use words and or pictures.

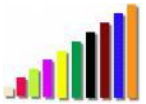
- Repeat this process for each of the 8 colours of rods.

The exact number of rods required to replace 22 rods is as follows:

Colour	Exact # of Rods
White	66
Red	33
Light Green	**22**
Purple	16 ½
Yellow	13 1/5
Dark Green	**11**
Black	9 3/7
Brown	8 ¼
Blue	7 1/3
Orange	6 3/5

In order to solve this problem, students must apply proportional thinking… such as ….

if 4 g = 3 p
then 8 g = 6 p
12 g = 9 p
16 g = 12 p
20 g = 15 p
24 g = 18 p



Ratio

A famous Texas chef has a secret recipe for her chili. When people ask her for it, she hands them a card with the complete recipe except for 1 detail. For the missing detail, she gives them the following clue: For a 3-quart pot of chili, use a combination of peppers and tomatoes totaling 12 items in one of the following ratios.

- If the answer is in whole numbers, how many peppers and how many tomatoes should you use?

A 

B 

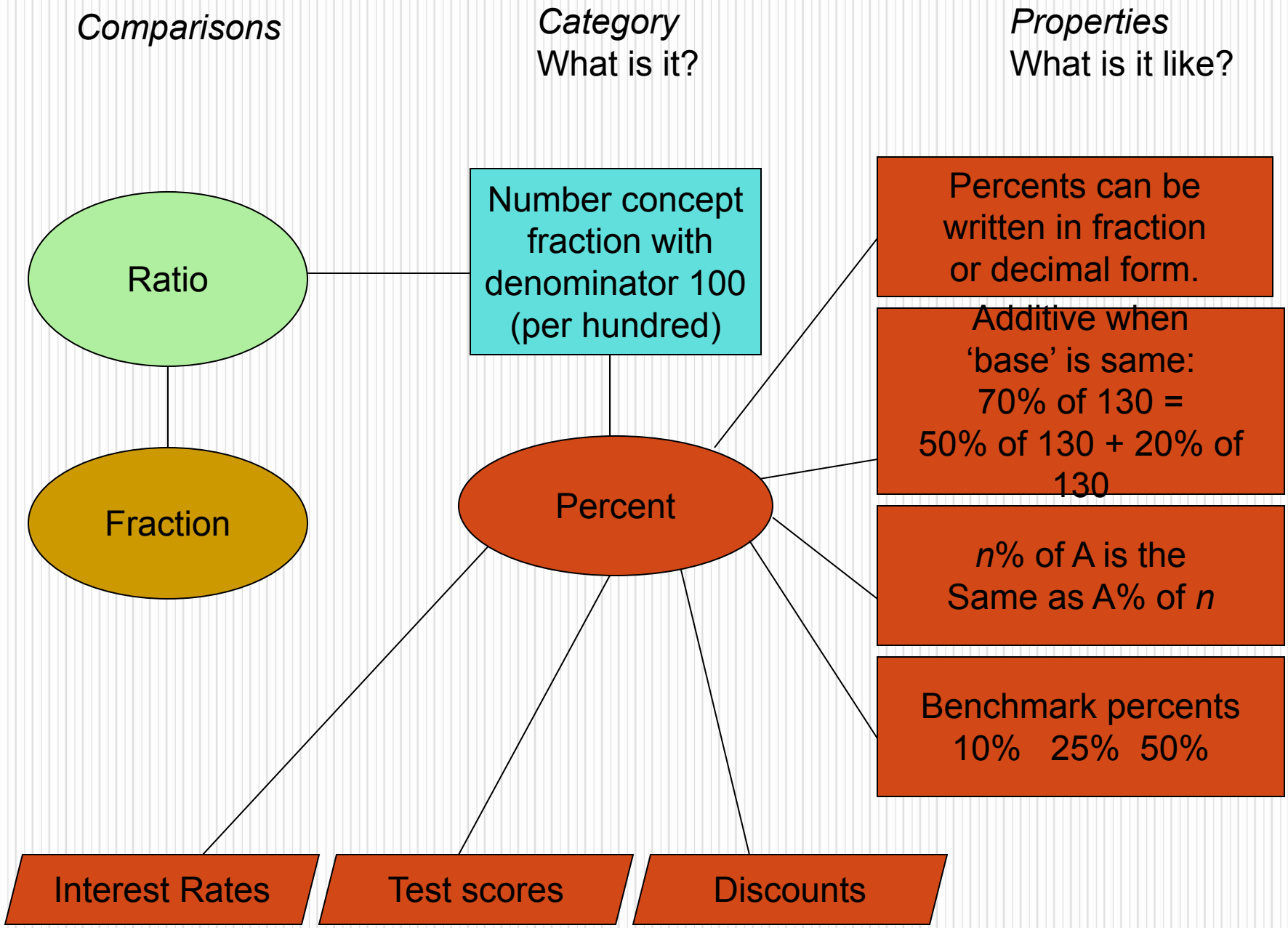
C 

Percents are a special sort of ratio…

… a ratio where the second number is 100.

- It can always be written as a decimal or vice versa
- Is based on the whole of which it is a percent
- They can be as low as 0%, but can go higher than 100%

"Making Math Meaningful to Canadian Students" Marian Small, Nelson


Comparisons
Category
What is it?
Properties
What is it like?
Ratio
Number concept fraction with denominator 100 (per hundred)
Percents can be written in fraction or decimal form.
Additive when 'base' is same: 70% of 130 = 50% of 130 + 20% of 130
Fraction
Percent
n% of A is the Same as A% of *n*
Benchmark percents 10% 25% 50%
Interest Rates
Test scores
Discounts

Exploring Percents with …

Base 10 blocks

Paving Places - Introducing

- Find an object in the room that costs about $1.00.
- This object 'did' cost $1.00 but it is now on sale at a 10% discount. How much does the item cost now?

$.90 RIGHT!

This is because $1.00 represents 100 cents and that 10% of 100 is represented by the fraction

10/100 or 1/10 and therefore

one tenth of 100 is 10.

If this item was on sale for 20% off,

how much would it cost?

Paving Places

- Completely tile, ‘pave’ a large sheet of paper (or desk top)
- You determine what kinds of blocks to use – according to the monetary values that have been assigned to each. They try to ‘pave’ with the least possible cost.
- **THINK:** If each unit ‘paver’ cost $3.00 what would be the cost of each long paver? Each flat paver?
- Estimate what would it cost to pave your workspace if you used unit pavers only? Record your estimate.

Paving Places – GOOD NEWS

SALE PRICES:

10% off on longs

20% off on flats.

- Now figure out how to pave your workspace the least costly way using any combinations of pavers.
- Record your work.
- Be ready to explain how you decided to pave your workspace.

Scratch and Save Event

A department store in Calgary is offering a scratch and save event in which you get to scratch 3 different amounts and use all three (imagine that!). You scratched 10%, 25% and 40%. You found the perfect shirt for work. Its original price is $40 Does it makes a difference which order the cashier applies the savings?

A record store in Calgary is offering a scratch and save event in which you get to scratch 3 different amounts and use all three (imagine that!). You scratched 5%, 10% and 20%. You found the perfect CD. Its original price is $10 Does it makes a difference which order the cashier applies the savings?

Provide evidence to support your answer.

Mrs. Flo Wer is planting a garden. She wants to follow the plan below:

Flo wants four-tenths of the garden to be planted with geraniums.

Flo wants fifteen hundredths of the garden to be planted with marigolds.

Flo wants three-tenths of the garden to be planted with tulips.

Flo wants the remaining section of the garden to be planted with sunflowers and daisies.

Complete the following chart:

Why do the students gardens look different? Are they correct? Explain your position.

Flower	Fraction	Decimal	Percent
Geranium			
Marigold			
Tulip			
Sunflo			

Questions

We are usually convinced more easily by reasons we have found ourselves than by those which have occurred to others.

Blaise Pascal (1670)

Resources cited in presentation

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