

Patterning and Algebraic Thinking

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Key Ideas in Patterns and Algebra

1. Patterns represent identified regularities based on rules describing the patterns' elements.
2. Any pattern can be represented in a variety of ways. Representing aspects of a situation with numbers make it easier to see patterns in the situation.
3. To describe a number pattern means to provide a precise rule that produces the pattern.
4. There are strategies that help us become better at recognizing common types of patterns. Data can be arranged to highlight patterns and relationships.
5. Patterns underlie mathematical concepts and can also be found in the real world. Our numeration system has a lot of specially built-in patterns that make working with numbers easier.
6. Some numbers have interesting or useful properties. Investigating the patterns in these special numbers can help us to understand them better.

Key idea

1

Patterns represent
identified regularities
based on rules describing
the patterns' elements

- ❖ The answer is....

Ask students to make up as many addition sentences that has a specified answer such as 24. After a period of time, ask specific students to share their work. Find those students who have put their sums in order.....

$$1 + 23 = 24$$

$$2 + 22 = 24$$

$$3 + 21 = 24$$

Ask what do you notice?

- ❖ I worked out the problem

$$237 + 492 = 729$$

Can you find other pairs that must add to 729 without doing the calculations?

- ❖ Try this problem

$$48.34 + 72.63 =$$

- ❖ Students can look in books or on the internet for patterns in nature.



- ❖ Ask: What makes this snake skin design a pattern?

- ❖ Show students part of a repeating Core pattern using blocks
- ❖ Ask students to predict what they will see as you uncover the rest of the pattern

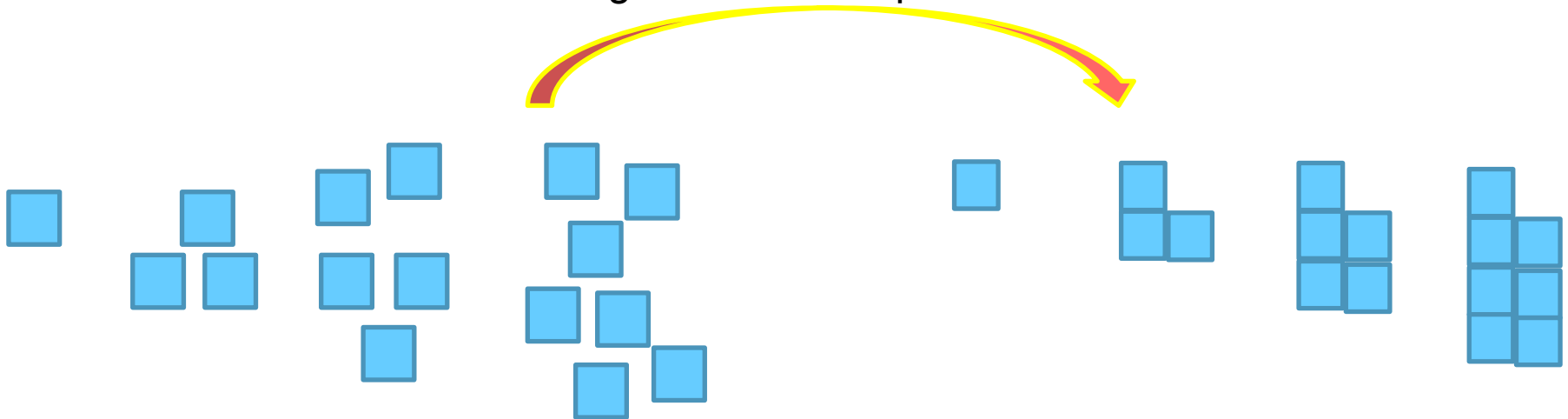


In order to focus on Key Idea 1 ask, *“Why did I need to tell you that it was a pattern for you to predict the next bead?”*

Provide students with 30 linking cubes all the same colour.

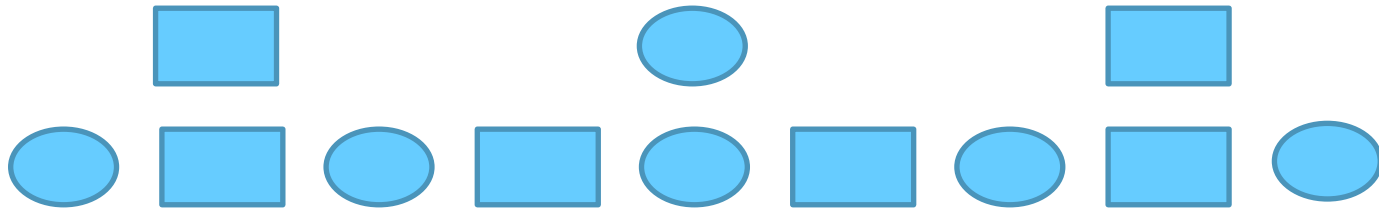
Use the cubes to show enough of a growing pattern so that another student could figure out the pattern.

Rearrange to make the pattern more obvious



To bring out Key Idea 1, ask:

What will your partner say repeats (or stays the same in your growing pattern?)



What do you notice about this pattern?

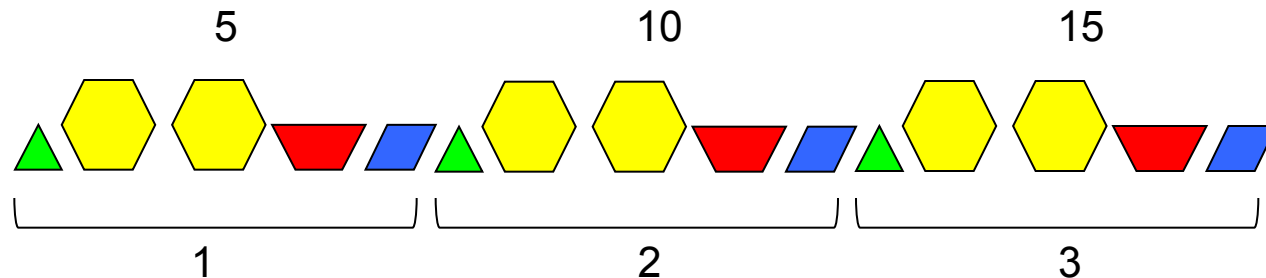


What do you notice about this pattern?
These are complex multi-attribute pattern

To bring our Key Idea 1 ask,
“What is it that makes the above patterns?”

Patterns can be repeating or increasing/decreasing

Learning Tasks



- a) What would the 20th shape be?
- b) What would the 30th shape be?
- c) What would the 32nd shape be?

Predicting Patterns

Making the link between repeating and increasing patterns

Key idea

2

Any pattern can be represented
in a variety of ways.

Representing aspects of a
situation with numbers make it
easier to see patterns in the
situation.

I am thinking about an ABCA pattern

- Use pattern blocks to show what your pattern might look like.

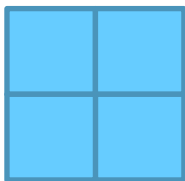
To bring out Key Idea 2 ask, “How are all of our patterns alike? How are they different?”

**Using linking cubes
represent the pattern 1, 4, 9,**

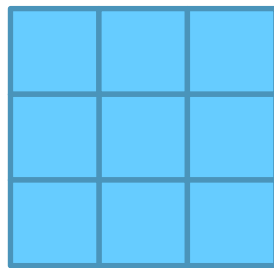
Using linking cubes represent the pattern 1, 4, 9,



1×1



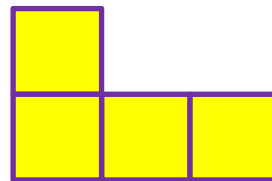
2×2



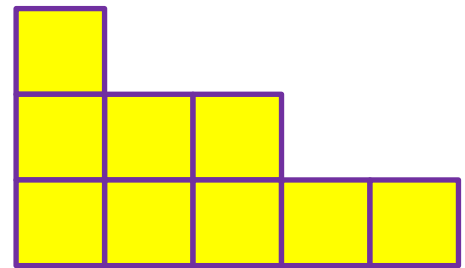
3×3



1



$1 + 3$



$1 + 3 + 5$

To bring out Key Idea 2 point out that the same pattern is represented in different ways.

One represents the squared nature of the numbers, while the other emphasizes that it is the sum of increasing odd numbers.

Ask, “Can you find another representation for the same number pattern?”

- ❖ You need to help a tricycle manufacturer work out how many parts are needed for different sized orders.
Begin with wheels. How many wheels will be needed for an order of one tricycle, two tricycles, three tricycles? How many wheels will be needed for nine tricycles?
- ❖ Produce a table showing the number of parts for different numbers of tricycles.....
'wheels', 'seats', hand-grips', 'tires'
- ❖ Draw out that the same number pattern may apply to different parts.

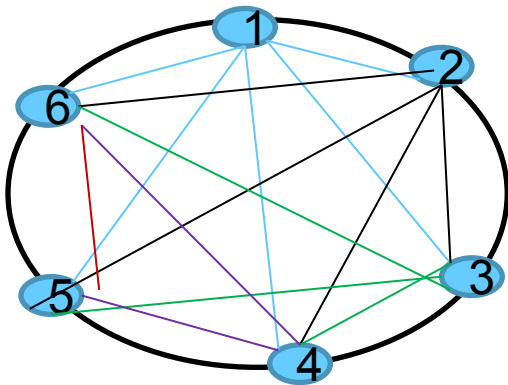
Learning Task – Handshake Problem

Six students arrive at a special gathering for students taking part in a Mathematics Fair. As the students are all from different schools, the teacher wants each to get to know the others. The teacher asks each student to shake hands with each of the other students and introduce themselves. How many handshakes took place?



Learning Task – Handshake Problem

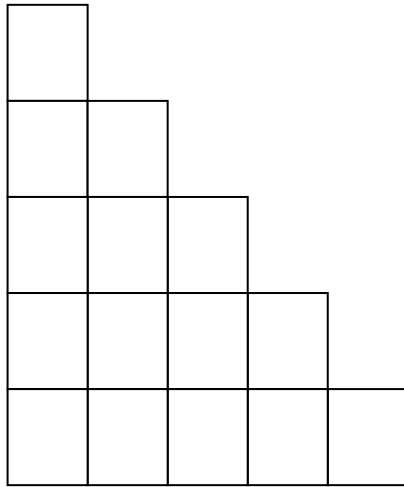
Models become tools for thinking



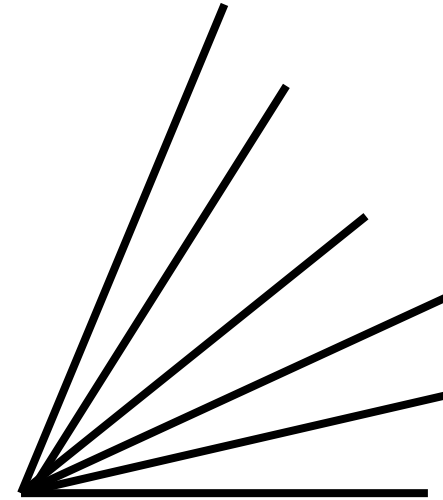
Elli-Fred	Fred-Gord	Gord-Hank	Hank-Indy	Indy-Jack
Elli-Gord	Fred-Hank	Gord-Indy	Hank-Jack	
Elli-Hank	Fred-Indy	Gord-Jack		
Elli-Indy	Fred-Jack			
Elli-Jack				

	1	2	3	4	5	6	7	8	9	10
1	x									
2		x								
3			x							
4				x						
5					x					
6						x				

How are these problems the same
and
how are they different from the
handshake task?



How many steps are there?



How many angles do you see?

$$5+4+3+2+1 = 15$$

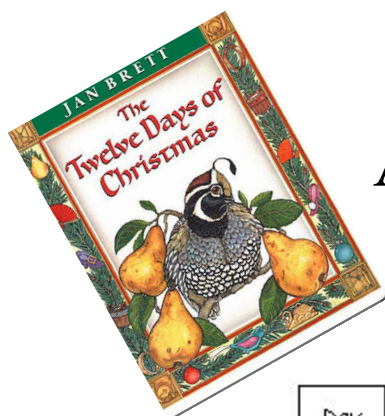
The Twelve Days of Christmas

How many gifts, in all, were given?

See if you can find a system and pattern to help.

Record your work and be able to explain how you arrived at your answer.

Extension : How much would these gifts cost?
Would they all fit in your bedroom?



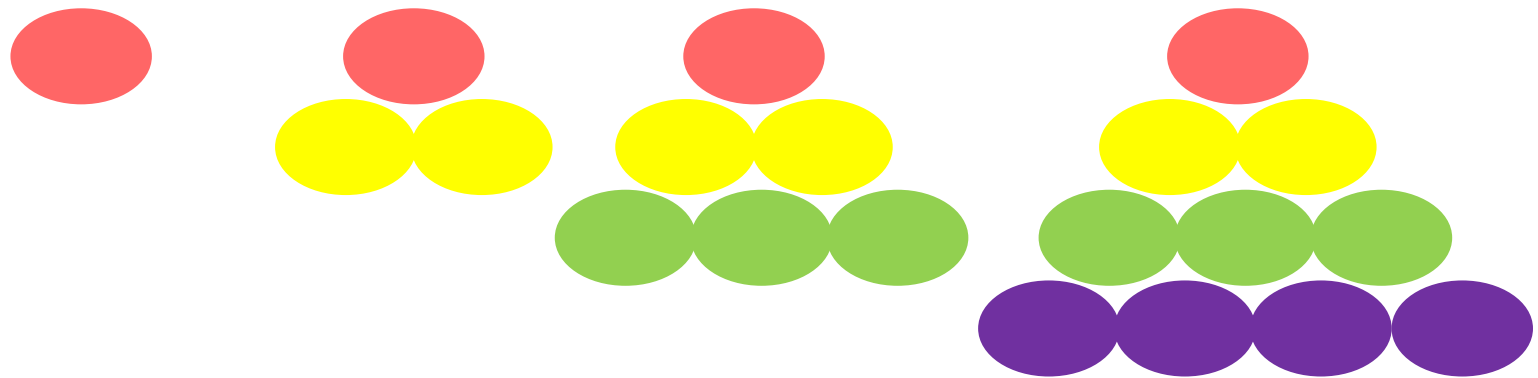
Twelve Days of Christmas

Analyzing Recursive Patterns

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10	Day 11	Day 12	Total each gift	
Partridge	1	1	1	1	1	1	1	1	1	1	1	1	12	10
Turtle Dove		2	2	2	2	2	2	2	2	2	2	2	22	8
French Hen			3	3	3	3	3	3	3	3	3	3	30	6
Calling Bird				4	4	4	4	4	4	4	4	4	36	4
Golden Rings					5	5	5	5	5	5	5	5	40	2
Geese						6	6	6	6	6	6	6	42	0
Swans							7	7	7	7	7	7	42	2
Maids								8	8	8	8	8	40	4
Ladies									9	9	9	9	36	6
Lords										10	10	10	30	8
Pipers											11	11	22	10
Drummers												12	12	
Total Gifts	1	3	6	10	15	21	28	36	45	55	66	78	364	

Note: From Analyzing Students' Thinking on Mathematical Tasks: Professional Development for Elementary Teachers (p. 34), by K. Willson, L. Gibeau and R. McKay, 2006, Edmonton, AB: Ioncmaste, University of Alberta. Copyright 2006 by the authors. Reprinted with permission.

Triangular numbers



$$\begin{array}{ccccccc} 1 & & 1+2 & & 1+2+3 & & 1+2+3+4 \\ 1 & + & 3 & + & 6 & + & 10 \end{array}$$

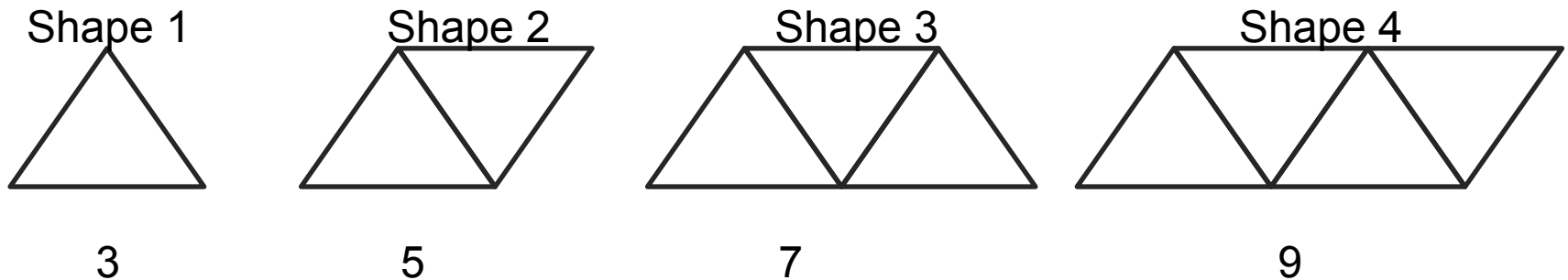
Total Gifts	1	3	6	10	15	21	28	36	45	55	66	78	364
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Key idea

3

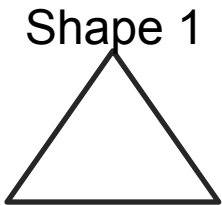
To describe a number pattern means to provide a precise rule that produces the pattern

Find a rule to describe the toothpick pattern....



- Write a rule to say how the toothpicks change with each new shape.
- Exchange rules with a partner and use the rule to find the number of toothpicks in the next few shapes.
- Ask, *“Did the rule work? Would your partner be able to predict the number of toothpicks for any shape position? Why or why not?”*

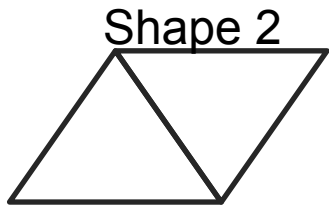
Can both of these students be right?



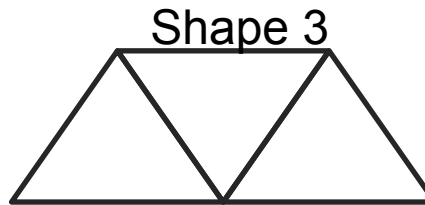
3

It's three, add the number of shape less one, times two

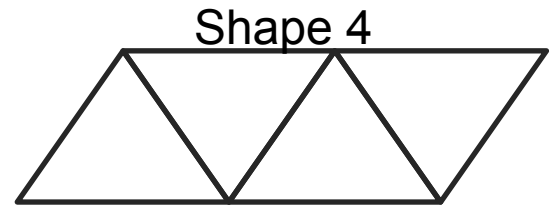
$$\begin{aligned}3 + (s-1) \times 2 \\3 + 2(s-1) \\3 + 2s - 2 \\2s + 1\end{aligned}$$



5



7



9

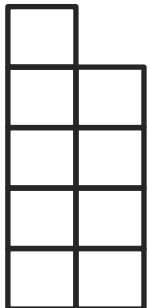
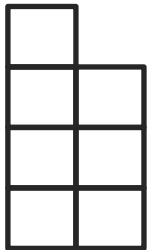
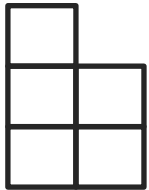
Times the number of the shape by two, then add one



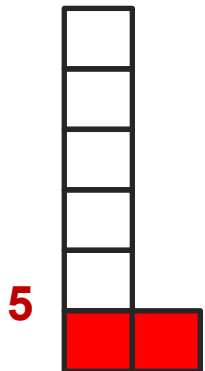
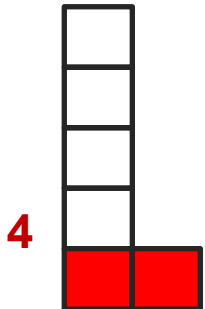
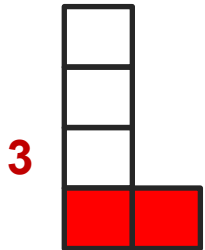
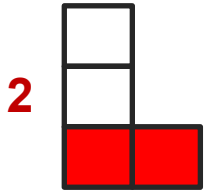
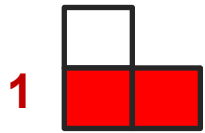
$$2s + 1$$

Build five stages of the following pattern using two different coloured tiles or blocks

What do you see as growing and what do you see as staying the same?

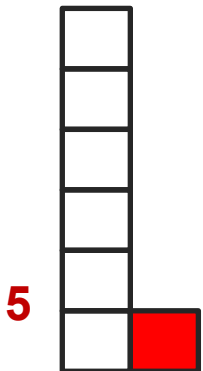
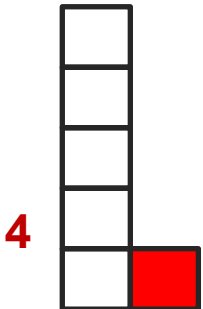
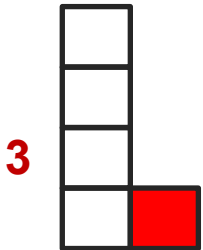
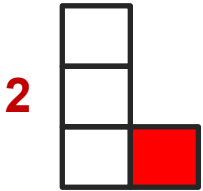
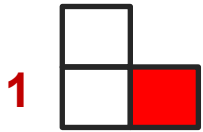


Decide on a pattern rule for stage 10 of this growing pattern.



Stage	What I see	Total
1	$2 + 1$	3
2	$2 + 2$	4
3	$2 + 3$	5
4	$2 + 4$	6
5	$2 + 5$	7

I see the bottom two red
staying the same
For stage 10, I see
 $2 + \text{stage number}$
 $2 + 10 = 12$



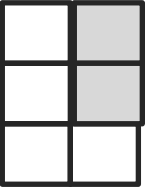
Stage	What I see	Total
1	$1 + 2$	3
2	$1 + 3$	4
3	$1 + 4$	5
4	$1 + 5$	6
5	$1 + 6$	7

I see the bottom one red
staying the same.
For stage 10, I see one red tile
plus a tower of one more than
the stage number.

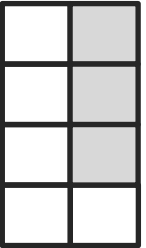
1



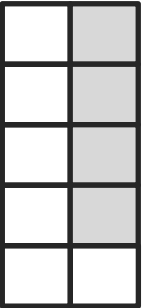
2



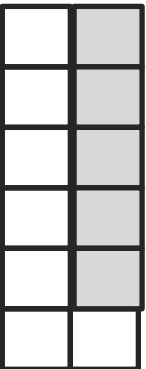
3



4



5



Stage	What I see	Total
1	$(2 \times 2) - 1$	3
2	$(2 \times 3) - 2$	4
3	$(2 \times 4) - 3$	5
4	$(2 \times 5) - 4$	6
5	$(2 \times 6) - 5$	7

2 is the base. I made a rectangle that was one more than the stage number. Then subtract the stage number.

For stage 10,
 $2 \times (10+1) - 10$

Consolidating ideas.

Sample A

For stage 10, I see
2 + stage number
 $2 + 10 = 12$

$$\underline{T = 2 + s}$$

Sample B

For stage 10, I see one
red tile plus a tower of
one more than the
stage number.

$$T = 1 + (s + 1)$$

$$T = 1 + s + 1$$

$$\underline{T = 2 + s}$$

Sample C

For stage 10,
 $2 \times (10 + 1) - 10$

$$T = 2(s + 1) - s$$

$$T = 2s + 2 - s$$

$$\underline{T = 2 + s}$$

What is the same and what is different about the formulas?

What if the total number of tiles was “64” what is the stage number?

Key idea

4

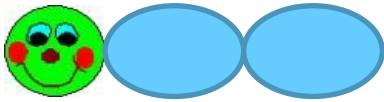
There are strategies that help us become better at recognizing common types of patterns. Data can be arranged to highlight patterns and relationships.

- Have students use the constant function on a calculator to generate patterns such as:
 'add 2' ($3 + 2 = = =$)
 'subtract 3' ($24 - 3 = = =$)
- Let them choose their own starting number and generate a sequence using the calculator.
- Try starting with 24 and using $\times 0.5$
- Invite students to investigate 'halving' sequences by giving them a strip of paper between 30 - 40 cm long. Have them fold it in half and measure it. Repeat . Have students compare their sequences.
- *What is happening to the numbers of each?*
- *How could you generate this sequence with a calculator?*
- *How is this different from a subtraction sequence?*

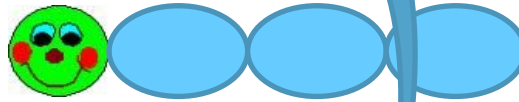
You may graph all these results

Worm

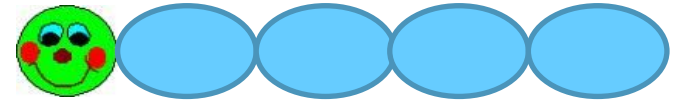
Recursive rule increase by '1'



Birthday 1



Birthday 2



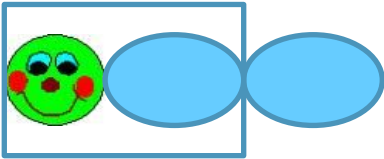
Birthday 3

Birthday	Number of body parts
1	3
2	4
3	5
4	6

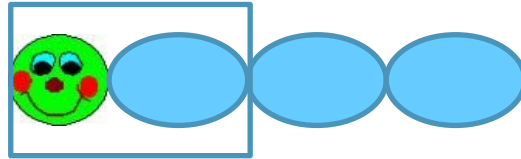
Explicit rule

$$T = 1s + 2$$

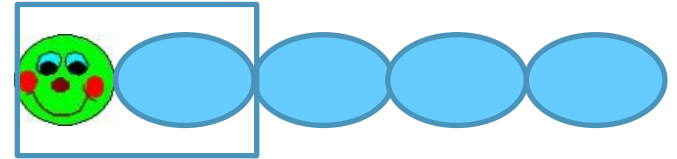
What is staying the same?



Birthday 1



Birthday 2

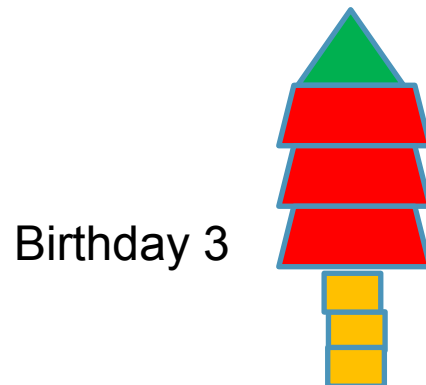


Birthday 3

Birthday	Number of body parts
0	2
1	3
2	4
3	5
4	6

You may graph this on your graph paper

Growing Trees



Birthday	# blocks
0	1
1	3
2	5
3	7

What is the recursive rule?
(how much each shape is
increasing by)

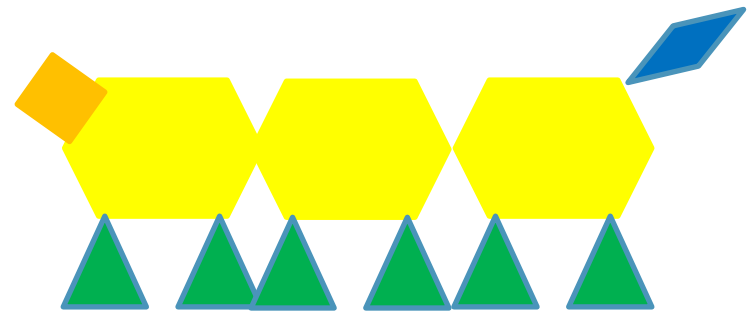
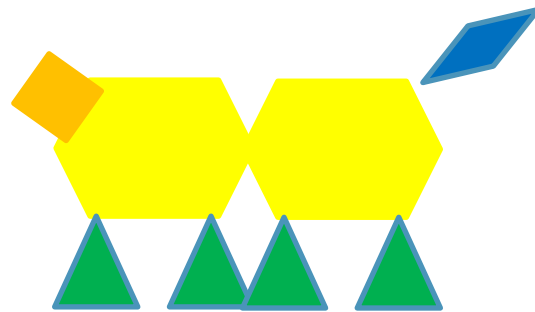
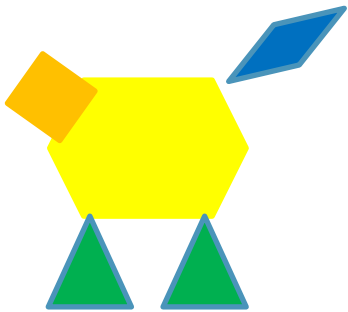
Increase by '2'

What is the explicit rule?
(the formula).

$$T = 2r + 1$$

You may graph this on your graph paper

Your turn – Growing Creatures



What is the recursive rule?
(how much each shape is
increasing by)

Increase by '3'

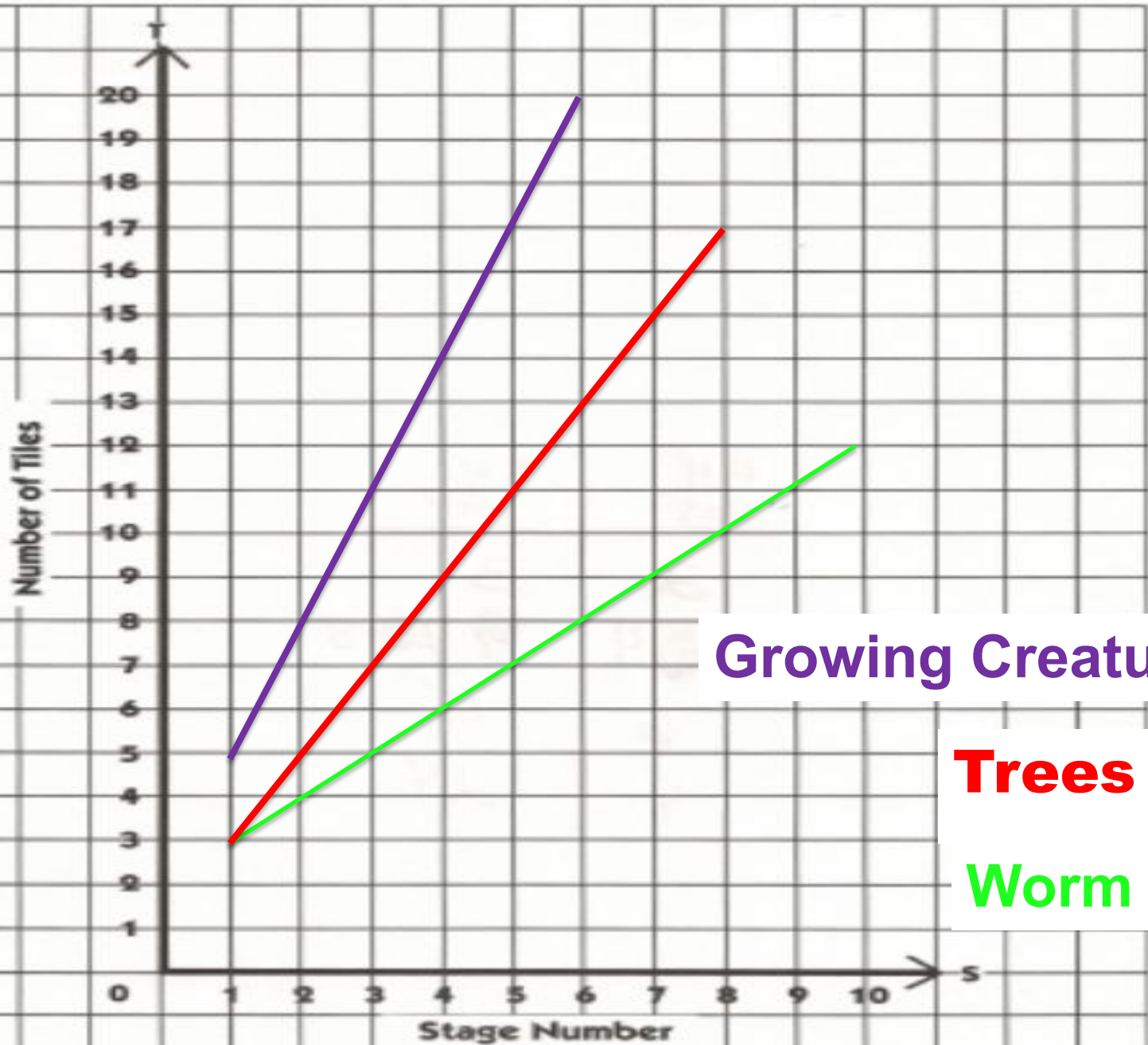
What is the explicit rule?
(the formula).

$$T = 3r + 2$$

Stage #	# of blocks
0	2
1	5
2	8
3	11

You may graph this on your graph paper

Tile Patterns



Growing Creatures

Trees

Worm

Key idea

5

Patterns underlie mathematical concepts and can also be found in the real world. Our numeration system has a lot of specially built-in patterns that make working with numbers easier.

- Create a number where you would say each of these words as you read the number:
Forty, five, million, thousand, two, hundred, six
- Write it symbolically.

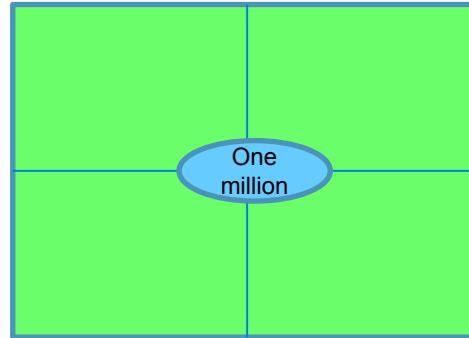
possible solutions:

45 600 002 or 46 502 000 or 5 206 042

How did you know your number would have at least 7 digits?

How did you know the digit 4 would be the middle digit in a period?

- In groups of 4, create a place-mat. In the centre, write the word “one million.”



- For two minutes, write about one million in your section of the place-mat.

Which of the things you wrote show that you understand the place value system?

Which of them show that you have compared one million to other benchmark numbers? Which did both?

How is this useful as an assessment for learning tool?

Why is it better to write “one million” than 1 000 000?

Follow-up questions

Ask students to think about:

- How many loonies make \$1 000 000?

How many \$100 bills?

- How long would a line of 1 000 000 pennies be?

- How long would it take to roll 1 million pennies?

- Then ask: *What place value ideas did you use to help you answer these questions?*

When a book of unexplainable occurrences brings Petra Andalee and Calder Pillay together, strange things start to happen: seemingly unrelated events connect, an eccentric old woman seeks their company, and an invaluable Vermeer painting disappears. Before they know it, the two find themselves at the center of an international art scandal, where no one — neighbors, parents, teachers — is spared from suspicion. As Petra and Calder are drawn clue by clue into a mysterious labyrinth, they must draw on their powers of intuition, their problem-solving skills, and their knowledge of Vermeer. Can they decipher a crime that has left even the FBI baffled?

Chasing Vermeer

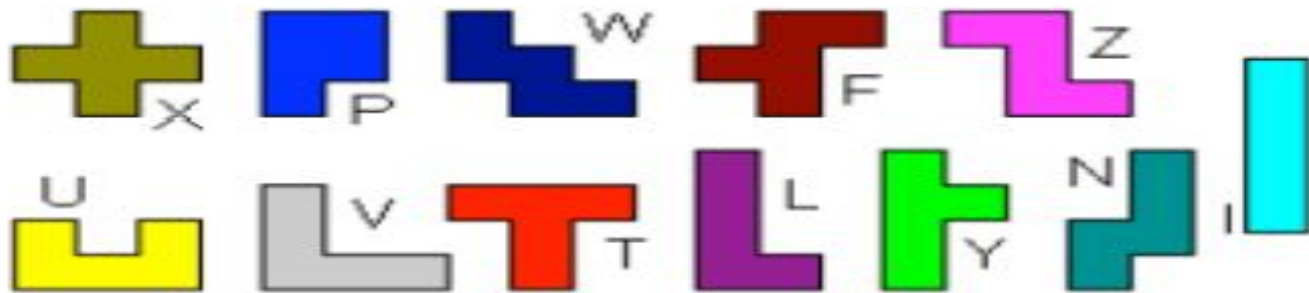
By Blue Balliett



Pentominoes

- Work with a partner to discover all the ways you can link 5 cubes at a time.
- Record your work on grid paper.
- When you think you have found all the arrangements, figure out the area and perimeter of each shape.

http://www.scholastic.com/blueballiett/games/pentominoes_game.htm



Shape	Area	Perimeter
X	5	12
P	5	10
W	5	12
F	5	12
Z	5	12
U	5	12
V	5	12
T	5	12
L	5	12
Y	5	12
N	5	12
I	5	12

Hide the numbers

Provide various pentominoes – each made of five squares as those of a 100 chart. Have students work in pairs and use a pentominoe to cover parts of the 100 grid. Say what numbers are hidden.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18		20
21	22	23	24	25	26	27	28		30
31			34	35	36	37			40
41	42		44	45	46	47	48	49	50
51			54	55	56	57	58	59	60
61	62	63	64	65	66		68	69	70
		73	74	75	76			79	80
81		83	84	85			88	89	90
91			94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110

Ask: How did you decide which numbers were hidden? What patterns did you use? How did the way our numbers are written help?

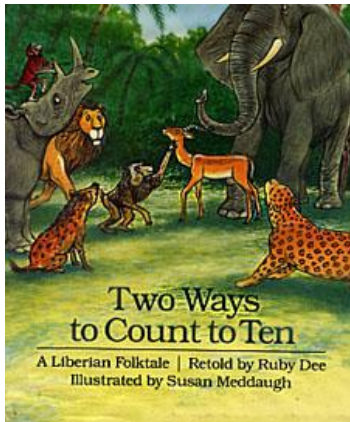
Key idea

6

Some numbers have interesting or useful properties. Investigating the patterns in these special numbers can help us to understand them better.

Ways to Count to Ten

A Liberian Folktale by Ruby Dee



- Leopard, who is king of all the other animals, holds a contest to find a successor. The challenge: throw a spear into the air and count to ten before it hits the ground. No matter how strong the animal is he can't throw the spear high enough or count quickly enough.

- Have the children predict who they think will be the king of the jungle.
- Read the story and as a group, discuss the ways the animals counted to ten. Record on the board and share how this is recorded.
 - For example $10 = 1, 2, 5, 10$
 - These are factors of 10.
 - How might we write it another way?
- Working cooperatively, in groups of 2, have the children discover all the ways to get to 1, 2, 3,30.
- Encourage them to record so that the information is meaningful to them.

** This activity may uncover the following mathematical ideas, depending on what patterns the children discover.**

prime numbers, composite numbers, square and triangular number sequences, factors, rules of divisibility and more...

Number	Factors	Ways
1		
2		
3		
4		
5		
6		
7		
8		
9		
10	1,2,5,10	4
11		

Number	Factors	Ways
1	1	1
2	1,2	2
3	1,3	2
4	1,2,4	3
5	1,5	2
6	1,2,3,6	4
7	1,7	2
8	1,2,4,8	4
9	1,3,9	3
10	1,2,5,10	4
11	1,11	2

Observations:

- Perfect squares have three factors. Is this always so? What do you notice about the next perfect squares? Can you generalize a statement about perfect square numbers?
- What do you notice about those numbers that only have two factors?
- If I gave you square tiles and grid-paper what kinds of rectangles could you make with those numbers? What do you notice?

Questions



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Resources

- Alberta Education – Patterns and Algebra K-3 and 4-6 workshops
- Small, M. Big Ideas from Dr. Small: Creating a Comfort Zone for Teaching Mathematics Grade K-3 Book, Nelson Education
- Small, M. Big Ideas from Dr. Small: Creating a Comfort Zone for Teaching Mathematics Grade 4-8 Book, Nelson Education
- Wickett, M. & Kharas, K & Burns, M. Lessons for Algebraic Thinking Grades 3 -5, Math Solutions Publications, Sausalito, CA