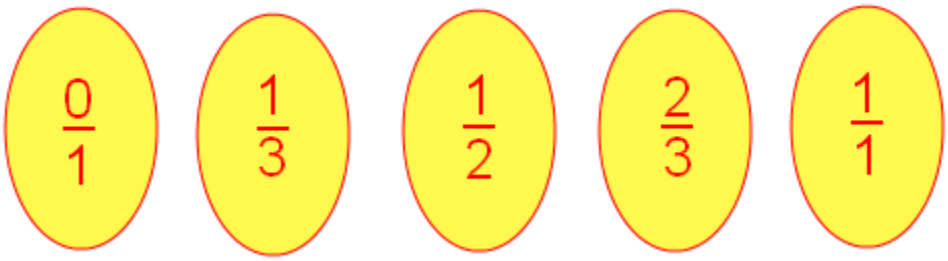
Farey Sequences

Ordering decimals can often be quite straightforward, but if I gave you a list of fractions, the task is often more difficult.

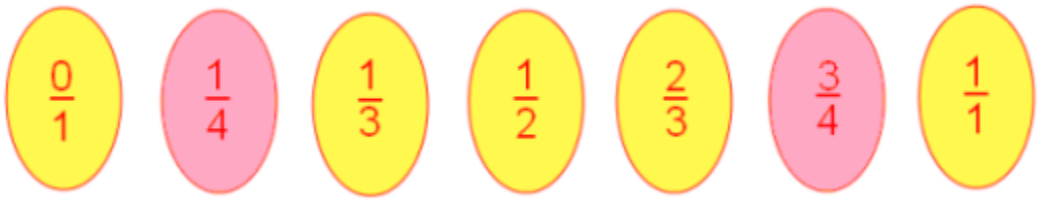
A geologist named John Farey investigated sequences of fractions in order of size – they are called Farey Sequences.

The third Farey Sequence, F_3, looks like this:



It lists the fractions between 0 and 1 in order, in their simplest forms, with denominators up to and including 3 (hence the 3 in the name of the sequence).

F_4 is shown below:



The new fractions to the list are shaded in pink.

With your group, discuss patterns that you observe in the numerators and denominators that could help you to predict F_5. Write these patterns down below. Write down F_5.

Which extra fractions are in F_5 that weren't in F_4? Make a note of these.

Use F_5 to help you complete F_6 and F_7.

Some further questions to think about:

There are lots of extra fractions added to F_{11} which are not in F_{10}.
There are only a few extra fractions in F_{12} which are not in F_{11}.
Can you explain why this is the case?
Can you predict when you will need a lot of extra fractions to get the next Farey Sequence?

Will every Farey Sequence be longer then the one before? How can you be certain?

Thus far, all of the Farey Sequences have contained an odd number of fractions. Can you find a Farey Sequence for which this is not the case?

In $F4$, 34 slotted in between 23 and 11. What do you notice about the fractions on either side when you slot in a new fraction?

Choose any three consecutive fractions from a Farey Sequence. Can you find a way to combine the two outer fractions to make the middle one?[1]

[1] Images taken, and information adapted from http://nrich.maths.org/2086/index?nomenu=1, A University of Cambridge endeavour.