Stage 1 – Desired Results

Established Goal(s):

- 7N3 - Demonstrate an understanding of the relationship between positive terminating decimals and positive fractions and between positive repeating decimals and positive fractions.
- 7N4 - Solve problems involving percents from 1% to 100%.
- 7N7 - Compare and order positive fractions, positive decimals (to thousandths) and whole numbers by using:
  - benchmarks
  - place value
  - equivalent fractions and/or decimals.

Process Skill(s):	**Understanding(s):**
✓ Communication (C) ✓ Connections (Cn) ✓ Problem Solving (PS) ✓ Mental Math and Estimation (ME) ✓ Reasoning (R) ✓ Technology (T) ✓ Visualization (V)	*Students will understand that...* ✓ Renaming fractions is often the key to comparing them or computing with them. Every fraction can be renamed in an infinite number of ways. ✓ Decimals are an alternative representation to fractions, but one that allows for modeling representations, comparisons and calculations. ✓ Percents, just like fractions and decimals, are comparisons of quantities. ✓ Percent always compares a quantity to 100.
Students will know... Terminology ✓ Numerator ✓ Denominator ✓ Percent ✓ Place Value ✓ "of" vs. "out of" ✓ Repeating Decimal ✓ Terminating Decimal ✓ Improper Fraction ✓ Mixed Number	*Students will be able to...* ✓ I can calculate a percentage. ✓ I can express percents as fractions and decimals. ✓ I can calculate percent of a number. ✓ I can use percent calculations appropriately in problem situations (such as, sales tax, discounts, tips, total costs, percent increase and decrease, etc.) ✓ I can express fractions as decimals. ✓ I can express terminating decimals as fractions. ✓ I can express repeating decimals as fractions. ✓ I can write a repeating decimals using bar notation. ✓ I can determine when it is appropriate to round, and to what place value. I can compare whole numbers, fractions, and decimals. ✓ I can order whole numbers, fractions, and decimals. ✓ I can correctly place whole numbers, fractions, and decimals on a number line.

Stage 2 – Assessment Evidence

Performance Task(s):	**Other Evidence:**
- Christmas Shopping Project - Menu Activities from M. Burns	- Unit Exam (Build from computerized assessment bank.)

Stage 3 – Learning Plan

Introduction:

- Have the class brainstorm to build a mind map on the whiteboard containing everything they know about fractions, decimals, and percents. Use the information shared by students to create Frayer Models for each of these words. Use questions such as: What is a percent? What does the word percent mean? What does the symbol look like? Where do you see percents in the real world? How are percents related to fractions and/or decimals? Where do we see fractions in the real world? Where do we see decimals in the real world?
- Have student look at their last unit test and convert their raw score to a percent. Discuss how they did this.
- Use "Activity 1 – Corn Maze" as a springboard.
- Use "Sense or Nonsense" activity from Marilyn Burns' Percent Activities. Pages 292-293.

Mental Math and Estimation

- Use one sheet daily from the "Percent Warmups" file as warmup/opener activities throughout unit.

Percent, Decimal, and Fraction Conversions

- Watch video from United Streaming such as "Maths Mansion Episode 27: Percentigmole" http://player.discoveryeducation.com/index.cfm?guidAssetId=0355D4BC-C944-46B4-A502-B4CBF687AB10&blnFromSearch=1&productcode=US
- Pose problem: "Janie, Robby, Christine and Ned are siblings.

  Janie has two-fifths of a pie.
  Robby has 40% of a pie.
  Christine has 0.4 of a pie.
  Ned has none.

  The children's mom says that whoever has the most pie has to split their pie with Ned. Who has to share? How could they fairly share the pie?"
- Use "Activity 2 – Percent Puzzle"
- Use Problem #1 from "Math 7 Percent Problems". Instructions and solutions are in accompanying "Case Study: Linking Fractions, Decimals, and Percents Using an Area Model."
- Use "Activity 3 – Tangram Puzzles."
- Use "Activity 4 – Smartie Lab."
- Use "Activity 5 – Geoboard Areas."
- Use "Activity 6 – Sports Challenge."
- Use "What Percent is Shaded?" activity from Marilyn Burn's Percent Activites. Pages 293-294.
- Formative Assessment: Use "Percents Journal."

- Exit Card Questions
    1. Lisa and Lou are in different math classes at school. Lisa scored 24 out of 29 on her last exam and Lou scored 27 out of 34 on his last exam. Lou says that he got more questions right and is therefore smarter? Is Lou correct? Explain how you know.
    2. Who has the most candy? Explain how you know.

       Jack has 15% of a bag.
       Julie has 3/15 of a bag.
       John has 0.25 of a bag.

    3. Who owes the most money? Explain how you know.

       Kari owes three-quarters of dollar.
       Kevin owes 65% of a dollar.
       Katy owes 0.70.

Working with Repeating Decimals

- Compare the words "terminating" and "repeating." Ask the class what they think a terminating decimal would be and what a repeating decimal would be. Share a few examples. Use Frayer models to record student thoughts. Discuss the use of "Bar Notation." Complete several examples of how to use bar notation to represent repeating decimals. Create a Frayer model for this word as well.
- Use "Activity 6 – Repeating Decimals Exploration."
- Formative Assessment – Worksheet 4.2

Comparing and Ordering Percents, Fractions, and Decimals

- Use "Activity 7 – Comparing Decimals and Fractions."
- Formative Assessment: Senteo Question Set "Comparing and Ordering Fractions Question Set"

Calculating Percent of a Number

- Use problems #2 and #3 from "Math 7 Percent Problems."
- Use "Mental Calculations with Percents" activity from Marilyn Burn's Percent Activities. Pages 296-297.
- Use "Activity 8 – Percents Jigsaw" as a jigsaw activity for the class.
- Use "Activity 9 – Sales Tax and Discounts." You will also need the "Toy Price List."
- Formative Assessment: "Percent Word Problems." Each student chooses 15 of the 30 questions to complete with a partner.

Wrap-Up

- Re-visit the Frayer models that were started throughout the unit. Make any additions and revisions necessary.
- Formative Assessment: Use the Menu Activities (set up like centers) from the Marilyn Burns's Percent Activities. Pages 297-300.
- Use "Using Graphs to Build Understanding of Percents" activity from Marilyn Burn's Percent Activities. Pages 295-296.

Frayer Model for ______________

Definition	Characteristics
Examples with Visual Representations	**Non-examples**

Activity 1

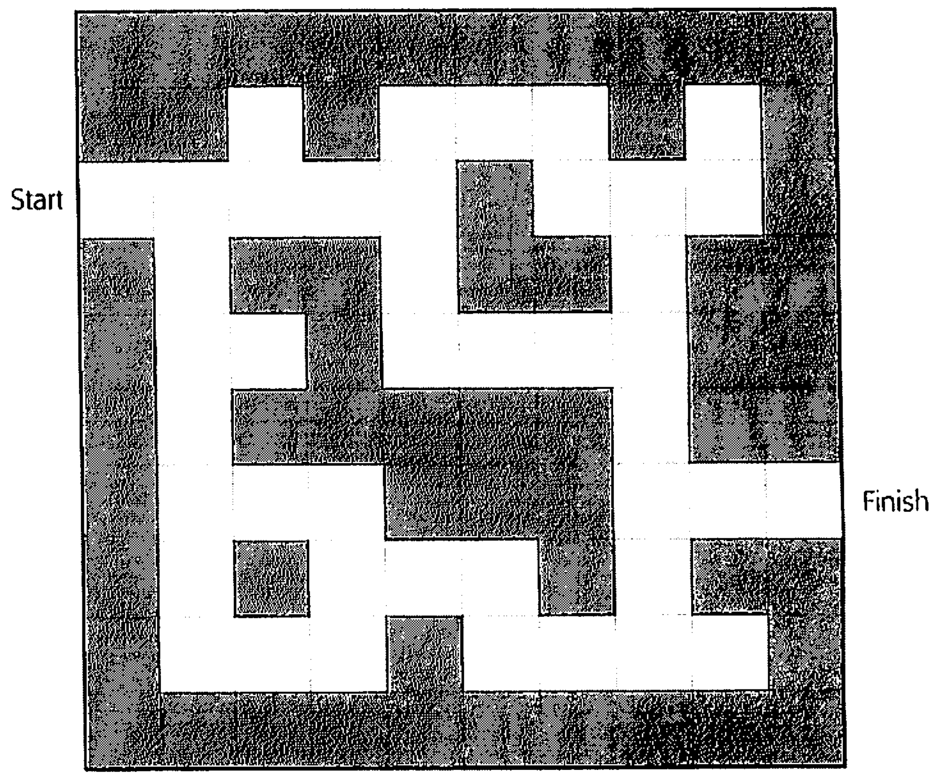
Chapter 4

Getting Started

YOU WILL NEED
- centimetre grid paper
- a calculator

Parts of a Whole

Some Canadian farmers build mazes in their cornfields to attract tourists. The following grid shows a plan for a simple maze in a square cornfield. The white squares are the paths.



How can you describe the paths in the maze using fractions, decimals, and percents?

A. What fraction of the square cornfield is used for the paths in the maze? Write an equivalent decimal.

B. What fraction of the cornfield is used for the corn? Write an equivalent decimal.

percent

a part to whole **ratio** that compares a number or an amount to 100; for example, $25\% = 25:100 = \frac{25}{100}$

ratio

a comparison of two quantities with the same units; for example, if you make juice using 1 can of concentrate and 3 cans of water, the ratio of concentrate to juice is 1:4, or 1 to 4.

C. What **percent** of the cornfield is used for the paths?

D. What percent of the cornfield is used for the corn?

E. Copy the maze onto grid paper, and trace a path from Start to Finish.

F. Count the squares on your path. Express the area of your path as part of the total area of the cornfield using fractions, decimals, and percents.

G. Explain why the areas of the paths in the maze are easily expressed as fractions, decimals, and percents of the cornfield's total area.

What Do You Think?

Decide whether you agree or disagree with each statement. Be ready to explain your decision.

1. If you get a score of 72% on a test, you must have had 72 questions right.
2. Your legs make up about 40% of your height.
3. 10% of an amount of money is not very much.
4. You can compare fractions, decimals, and percents on a number line.

Marilyn Burns - Percent Activities

Percents

Overview

Students have many experiences with percents before they study them formally in school. They know that a 50 percent sale means that prices are cut in half and that a 10 percent sale doesn't give as much savings. They understand what it means to earn a 90 percent grade on a test. They hear on TV that some tires get 40 percent more wear, that a tennis player gets 64 percent of her first serves in, that there is a 70 percent chance of rain tomorrow. Common to these sorts of experiences is that percents are presented in the context of situations that occur in students' daily lives.

The goal for instruction in percents should be to help students learn to use percents appropriately and effectively in problem situations. This means that, when given a situation that involves percents, students should be able to reason mathematically to arrive at an answer, to explain why that answer is reasonable, and to make a decision about the situation based on the answer.

The activities presented in this section are designed to build on what students already know and to help them extend their understanding of how to reason with percents. The ideas do not provide a comprehensive guide for teaching percents. Rather, they offer models of ways to introduce ideas about percents, suggest problem situations that engage students, and give alternative methods for assessing what students understand.

Whole-Class Lessons

Introducing Percents in Contexts

It makes sense for formal lessons on percents to build on what students already know. Instructional activities should expand students' knowledge from their daily life experiences into more general understanding.

Sense or Nonsense?

Students are to decide whether the statements are reasonable and why. You can discuss each of the statements in a whole-class discussion or have small groups of students discuss and write about them and then present their explanations to the class. For another class activity or for homework, have students write additional statements like these and use them for further class discussions.

1. Mr. Bragg says he is right 100% of the time. Do you think Mr. Bragg is bragging? Why?
2. The Todd family ate out last Saturday. The bill was $36.00. Would a 50% tip be too much to leave? Why?
3. Joe loaned Jeff a dollar. He said the interest would be 75% a day. Is this a pretty good deal for Joe? Why?
4. Cindy spends 100% of her allowance on candy. Do you think this is sensible? Why?
5. The Never Miss basketball team members made 10% of the baskets they tried. Do you think they should change their name? Why?
6. Sarah missed 10 problems on the science test. Do you think her percent is high enough for her to earn an A? Why?
7. Rosa has a paper route. She gets to keep 25% of whatever she collects. Do you think this is a good deal? Why?
8. The weather reporter said, "There's a one hundred percent chance of rain for tomorrow." Is this a reasonable prediction for this month? Why?
9. Ms. Green was complaining, "Prices have gone up at least two hundred percent this past year." Do you think she is exaggerating? Why?
10. A store advertised, "Best sale ever, 10% discount on all items." Is this a good sale? Why?

The Harper's Index Problem

As a regular feature, *Harper's Magazine* includes Harper's Index, a listing of statistical information about a variety of world issues (see *www.harpers.org/HarpersIndex.html*). An item in the October 1989 magazine read: "Percentage of supermarket prices that end in the digit 9 or 5: 80%."

Use this statement for a class investigation in which students collect and analyze data in order to test its validity. Ask students to bring supermarket receipts to class. In small groups, have students examine their receipts to see whether they support the statement. Ask groups to write a report about their investigation that includes the following:

data (shown in a neat and organized manner)
calculations (with an explanation about why the procedures they used make sense)
conclusion (explained clearly and concisely)

Have groups present their findings. Note: Students in one class raised the issue that while fruit, vegetables, and meat might have prices per pound that end in nine or five, the cost on the receipt won't necessarily reflect this for different amounts purchased. Therefore, they did a second study of their data in which they eliminated items in this category. The discussion about this gave the students experience with the kinds of difficulties that arise in real-life statistical investigations.

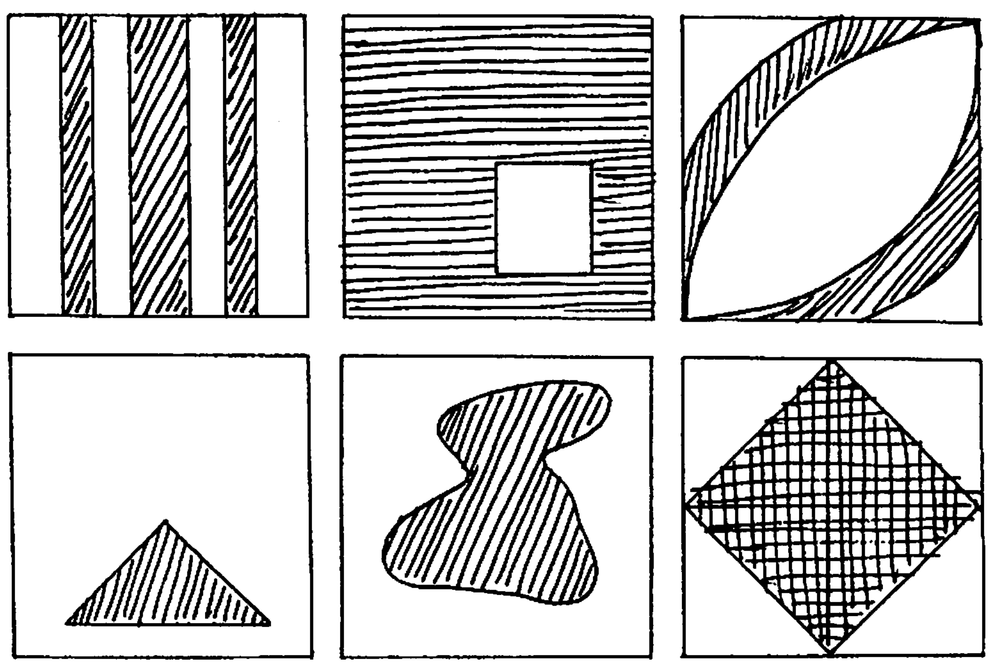
A Geometric Perspective on Percents

This activity presents students with a spatial model for thinking about percents. It introduces students to the idea that percents are parts of one hundred and gives them the opportunity to estimate areas and to express their estimates in terms of percents.

What Percent Is Shaded?

Make transparencies of the sheet of 10-by-10 grids (see Blackline Masters) and cut them apart so each student has one grid. Also, prepare a sheet of shapes such as the ones shown below.

Before distributing any of the materials, do several sample problems with the class. Draw a square on the board and shade part of it. Ask the students to estimate the percent of the square that is shaded. Have them



discuss this in small groups and then report their estimates and explain their reasoning. Do this for several shapes. Shade regions for which a range of responses is reasonable so that the emphasis is on students' reasoning, not on finding an exact answer that can be proved right or wrong.

Show students one of the transparent 10-by-10 grids. Ask them to discuss among themselves how to use the grid to estimate what percent of a square is shaded. It's important to introduce students to the idea that percent means "part of one hundred." Since there are one hundred small squares on the grid, the number of small squares that are shaded tells what percent of the grid has been shaded.

When you feel that students understand the task, distribute a worksheet of shapes and the transparent grids. Ask them to estimate the percent of each square that is shaded and use their transparent grids to check their estimates. Have students do this individually and then compare their answers in small groups. Have a class discussion about any they couldn't resolve.

Have students make up shapes for others to try. Having them put their shapes on index cards is an easy way to create a class set of problems.

Using Graphs to Build Understanding of Percents

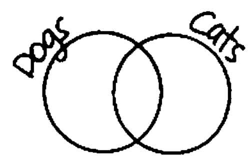
The questions that follow are ideas to be used for class graphs. The graphs suggested have been chosen so that students can compare the statistical result of their class samples with the percentages reported for the population at large. Use the graphs to initiate discussions about percents. For each, ask the following types of questions:

How many students indicated ____________?
Is this more or less than fifty percent?
About what percent of our class reported ____________?
How did you figure that percent?
The national statistic is ____________. How does our class sample compare with this national statistic?

The following statistics are from *American Averages*, by Mike Feinsilber and William B. Mead (1980).

1. Do you bite your fingernails? (Between 25 percent and 36 percent of college students do.)
2. How many hours a day do you sleep? (Sixty-three percent of adults sleep between seven and eight hours a day.)
3. Do you get a regular allowance? (Fifty percent of twelve- to seventeen-year-olds do.)

4. Do you view smoking as a sign of weakness? (Seventy-five percent of Americans do.)
5. Are you wearing athletic shoes or sneakers? (Twenty-five percent of shoes worn by Americans are athletic shoes or sneakers.)
6. Which do you generally take, a shower or a bath? (Fifty-nine percent of teenage girls and seventy-eight percent of teenage boys take showers rather than baths.)
7. What is your favorite animal? (Nine percent of children report that the horse is their favorite animal.)
8. What animals live in your home? (Thirty-three percent of U.S. households have a dog; 12.5 percent have a cat; 10 percent have both.)



9. Do fish live in your household? (Just 6.2 percent of households have fish.)
10. Have you ever taken piano lessons? (Forty percent of American children take piano lessons at some time or another.)
11. How many TVs are there in your home? (Fifty percent of households have two or more TVs.)
12. Which soft drink would you order: Coke, Pepsi, Dr. Pepper, 7-Up, other? (Twenty-six percent of Americans order Coke; 17 percent order Pepsi; the rest order something else when ordering a soft drink.)
13. Is there a child in your family under the age of six? (Twenty-five percent of American families have at least one child under the age of six.)
14. Is there a dishwasher in your home? (There are dishwashers in 40.9 percent of homes.)
15. How are the dishes done in your home most often: by hand or by a dishwasher? (In 58 percent of American homes, dishes are done by hand.)
16. Are you right- or left-handed? (Twelve percent of Americans are left-handed.)

Mental Calculation with Percents

It's useful for students to learn to calculate mentally with percents and to find strategies for doing so that make sense to them. Initiate a discussion about this by asking the students: "What is fifty percent of one hundred dollars?" Have volunteers respond, and ask each to explain how he or she knows that $50 is the correct answer. It's important for students to know that you value their thinking, not just their right answers. Don't stop after just one student's explanation, but have others also explain their reasoning.

Then ask: "What is twenty-five percent of one hundred dollars?" Again, have students give answers and explain their reasoning. Typically, fewer students are sure about this problem. If no one can explain, offer several ways for them to think about it: that 25 percent is half of 50 percent, so the answer is half of \$50.00, that there are four \$0.25 in \$1.00 and similarly there are four \$25.00 in \$100.00, that 25 percent is one-fourth of 100 percent and one-fourth of \$100.00 is \$25.00.

Continue the same sort of discussion for 10 percent, 5 percent, and 1 percent. Then write the following on the board:

100% of \$200 is ______.	*10% of \$200 is* ______.
50% of \$200 is ______.	*5% of \$200 is* ______.
25% of \$200 is ______.	*1% of \$200 is* ______.

Have students work in small groups to find the answers. Then have a class discussion in which they present their answers and reasoning.

Give the students other problems to solve by changing the \$200 to another number: \$500, \$300, or \$75, for example. Also, give the students problems such as the following:

Financing a loan costs approximately 11% a year. About what is the interest charge for a \$400 loan?

A poll reported that 97.6% of a town's voters support the mayor. The voter population of the town is 12,500. About how many voters support the mayor?

If dinner in a restaurant costs \$37.90, how much is a 15% tip?

Independent Activities [Menu]

Have students work on independent activities in pairs or small groups. Then have whole-class discussions in which students present their findings, the different methods they used, and the difficulties they encountered.

Percents in the Newspaper

You need: newspaper

Look for articles (not advertisements) in which percents are used. Choose one to present to the class. In your presentation, explain the meaning of the percents.

Figuring Tips in Restaurants

Read the following story.

Four people went to a restaurant for pizza. They ordered two medium pizzas: one pepperoni and one plain. They each ordered a soft drink. The waitress brought their drinks right away, which was good since they were all thirsty. The pizzas were ready soon afterward, and she brought them right over, with a stack of extra napkins for them to use. The bill was \$18.90 without tax.

Answer two questions:

1. What percent tip would you give?
2. How much money would you leave?

Explain your reasoning for each.

Write a restaurant story for others to solve.

Comparing Advertisements

You need: assortment of magazines or newspaper supplements that contain advertisements

Clip four advertisements that offer discounts for items and paste them on a sheet of paper. Include one that indicates the percent the customer will save, one that gives the sale price, and two of your choice. Decide which of the advertisements gives the customer the best deal. Record your decision on the back, including an explanation of your reasoning.

Trade with other groups and decide on the best deal for the advertisements they clipped. Compare your decisions.

The Photocopy Problem

You need: copies of a cartoon or drawing, including 1 full-size copy, 3 reductions, and 2 enlargements

Figure what percent was used for each reduction and enlargement. Record your solutions and explain the procedures you used.

The Discount Coupon

You need: discount hamburger coupon

About what percent discount do you get with this coupon? Show your work and explain why your method and answer make sense.

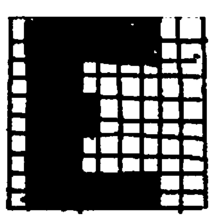
Find one or two other coupons or advertisements that could be used for the same sort of investigation. Figure the percent discount for each of them.

Try the investigation for coupons other students have found.

Block Letters

You need: 10-by-10 grids, 1 sheet (see Blackline Masters)

Make block letters on 10-by-10 grids, trying to make each one so it covers as close to 50 percent of the grid as possible.



The Warehouse Problem

For the following problem, explain your thinking and show all the work that helps make your explanation clear.

A warehouse gives a 20% discount on all items, but you also must pay 6% in sales tax. Which would you prefer to have calculated first—the discount or the tax? Does it matter?

Percent Stories

Write a percent story problem that follows two rules:

1. It must end in a question.
2. The question must require that percents be used to answer it.

On a separate paper, solve your problem. Show your work and include an explanation of why your method makes sense.

Exchange papers and solve each other's problems.

Assessing Understanding of Percents

Observing Students

During class discussions and when students are working on the independent tasks, circulate, observe, and, at times, question them about their thinking and reasoning. Look and listen for evidence of students' ability to calculate with percents and relate percents to contextual situations.

Individual Assessments

The following written assignments offer ways to find out about students' understanding of percents.

1. Before beginning instruction, ask students to write what they know about percents. Have them do this again at the end of the instructional unit or even midway through. Return papers to the students and ask them to reflect on their learning.
2. Assign problems such as those suggested for whole-class lessons. Ask students to present solutions in writing and to explain their reasoning.

TRANSPARENCY 2

Use Fractions to Find a Percent

$10\% = \frac{1}{10}$	$25\% = \frac{1}{4}$	$50\% = \frac{1}{2}$
$33\frac{1}{3}\% = \frac{1}{3}$	$20\% = \frac{1}{5}$	

To estimate the savings on the coat:

Think: $33\frac{1}{3}\%$ of \$120

$\frac{1}{3}$ of 120

\$40 off

TRY THESE

$\frac{1}{4}$

1. 25% of 80
2. $33\frac{1}{3}\%$ of 66
3. 50% of 48
4. $33\frac{1}{3}\%$ of 300
5. 25% of 36

$\frac{1}{2}$

6. 50% of 30
7. 25% of 24
8. 25% of 160
9. 50% of 440
10. $33\frac{1}{3}\%$ of 36

TRANSPARENCY 3

Estimate the Percent

Use a nice fraction for the percent, then estimate.

How are these estimates done?

$\frac{1}{4}$ 60

24% of 62

Estimate: 15

$\frac{1}{2}$ 90

49% of 89

Estimate: 45

TRY THESE
What nice fraction would you use?

1. 19% **2.** 34% **3.** 26% **4.** 12% **5.** 47%

Estimate.

6. 36% of 129

7. $24\frac{1}{2}$% of 51

8. 31% of 65

9. 54% of 133

10. 27% of 45

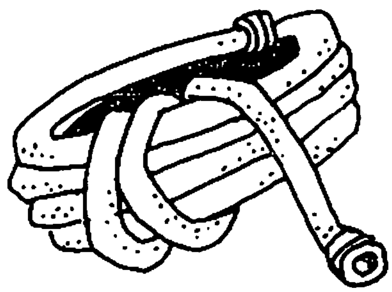
11. 48% of 119

12. 19% of $148\frac{1}{3}$

13. 23.4% of 61.3

TRANSPARENCY 4

What Percent do you Save?



Was: \$21
Now: \$5 off

% Saved: ____ %

$\frac{5}{21}$ is about $\frac{1}{4}$, and $\frac{1}{4}$ is 25%.

Was: \$119
Now: \$40 off

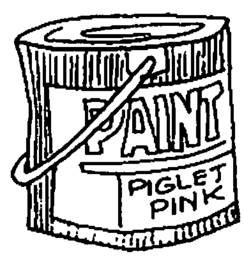
% Saved: ____ %

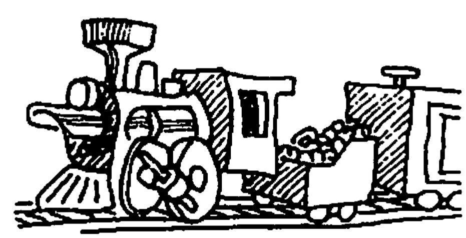
$\frac{40}{119} \approx \frac{40}{120}$, or about $\frac{1}{3}$.

TRY THESE

1. Was: \$19
   Now: \$2 off

   % Saved: ____ %



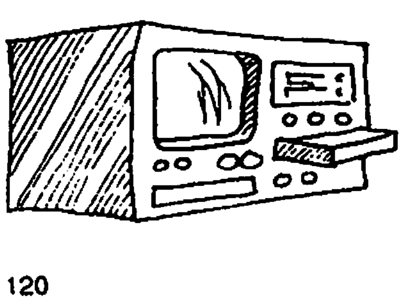


3. Was: \$78
   Now: \$24 off

   % Saved: ____ %

2. Was: \$498
   Now save \$95

   % Saved: ____ %



4. Was: \$59.95
   Now save \$19.95

   % Saved: ____ %

Using Fractions to Estimate Percents

Compute the exact answer in your head.

1. 1% of 234.6 = ______
2. 1% of 427 = ______
3. 10% of 842 = ______
4. 10% of 46.7 = ______
5. 1% of 4,682 = ______
6. 10% of 92.56 = ______
7. 25% of 36 = ______
8. $33\frac{1}{3}$% of 45 = ______
9. 50% of 0.250 = ______
10. 50% of 70 = ______
11. $33\frac{1}{3}$% of 600 = ______
12. 25% of 420 = ______

Rewrite each example by making the percents and/or numbers easy to compute mentally.

13. 34.7% of 60

    ______ of 60

14. 9.5% of 120

    ______ of 120

15. $48\frac{1}{2}$% of 250

    ______ of 250

16. $1\frac{1}{4}$% of 27

    ______ of 27

17. 26.7% of 158.92

    ______ of ______

18. 31% of 118.6

    ______ of ______

19. $22\frac{3}{4}$% of 47.6

    ______ of ______

20. 52.5% of 37

    ______ of ______

21. 11.5% of 134.62

    ______ of 134.62

Change one or both numbers in your head to estimate each problem.

22. $55\frac{1}{3}$% of 147.3

    ______ ESTIMATE

23. 0.92% of 326.4

    ______ ESTIMATE

24. 35% of 65.9

    ______ ESTIMATE

25. 11% of 106.2

    ______ ESTIMATE

26. 22.9% of $49\frac{1}{4}$

    ______ ESTIMATE

27. 9.86% of 27

    ______ ESTIMATE

28. $32\frac{1}{2}$% of 16.8

    ______ ESTIMATE

29. 1.1% of 4,560

    ______ ESTIMATE

30. 27.3% of 237.5

    ______ ESTIMATE

31. $\frac{7}{8}$% of 67

    ______ ESTIMATE

32. 33% of 72.8

    ______ ESTIMATE

33. 47.6% of 769

    ______ ESTIMATE

Using Fractions to Estimate Percents

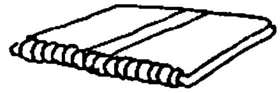
Estimate the savings on each item.

34. Snow Blower

Reg Price $645.89
35% OFF!

____
ESTIMATE

35. Notebook



Reg. Price $2.89
10% OFF!

____
ESTIMATE

36. Basketball

Reg. Price $15.95
50% OFF!

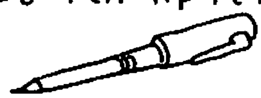
____
ESTIMATE

37. 14k Gold Necklace

Reg. Price $25.89
33 1/3% OFF!

____
ESTIMATE

38. Felt Tip Pen



Reg. Price $1.49
25% OFF!

____
ESTIMATE

39. Aluminum Cookware Set

Reg. Price $79.93
30% OFF!

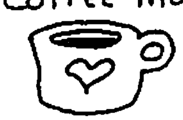
____
ESTIMATE

40. Folding Chair

Reg. Price $24.86
10% OFF!

____
ESTIMATE

41. Coffee Mug



Reg. Price $2.39
25% OFF!

____
ESTIMATE

42. Men's Sweater

Reg. Price $37.89
35% OFF!

____
ESTIMATE

Think it Through

When you **save** 25% on an item, you **pay** the remaining 75%.

Coat

$90 **25% off**

Save: $\frac{1}{4}$ of 90 → $22

Pay: $\frac{3}{4}$ of 90 → $66

1. Blouse: $36.95 — Save $33\frac{1}{3}$%

   Save: $\frac{1}{3}$ of $36.95 __________

   Pay: $\frac{2}{3}$ of $36.95 __________

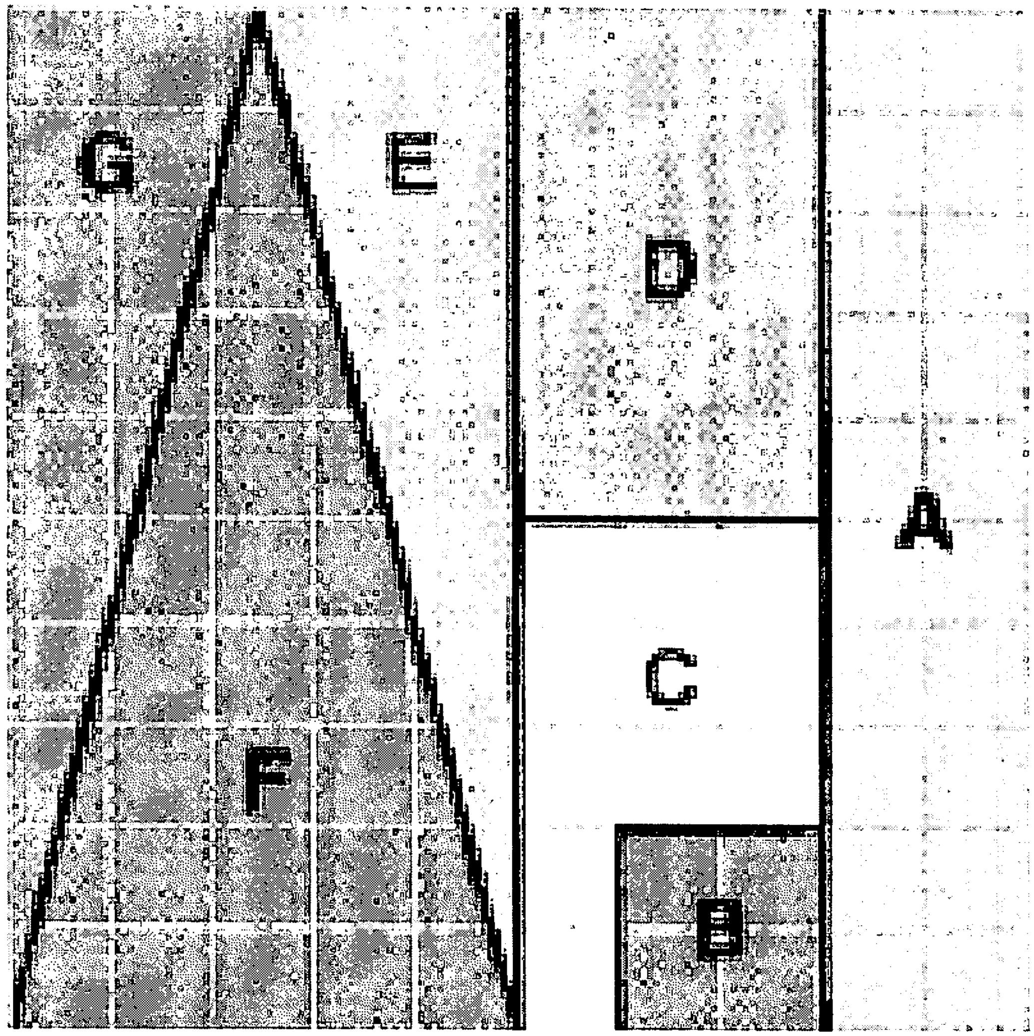
2. Sofa: $789 — Save 25%

   Save: $\frac{1}{4}$ of $789 __________

   Pay: $\frac{3}{4}$ of $789 __________

Activity 2: Percent Puzzle

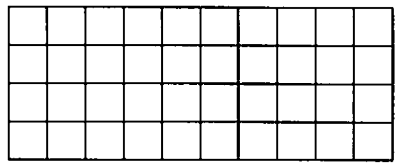
Describe each section of the puzzle as a percent, fraction, and decimal.



	Percent	Fraction	Decimal
A			
B			
C			
D			
E			
F			
G			

Math 7
Fraction, Decimal, Percent Problems

1. Shade 6 of the small squares in the rectangle shown below.



Using the diagram, explain how to determine each of the following:

a) The percent of the area that is shaded.
b) The decimal part of the area that is shaded.
c) The fractional part of the area that is shaded.

2. Why Isn't Pat Broke?

> The net worth of Pat's Candy Company at the start of a particular year was $32,000.00. Because of declining sales, Pat's company began losing 10 percent of its worth each month. At the end of the year, the company was still worth $9,037.74. How can that be? Wouldn't losing 10 percent per month for just ten consecutive months, let alone twelve, leave Pat with a worth of $0?

Why Isn't Pat Broke?

Discussing the problem shown in **figure 2** will help clarify the truth that 10 percent of a dwindling amount is an ever-dwindling amount. Ten percent of something remains a consistent size only if the original amount under consideration does not vary. The teacher may want to ask students this series of questions:

1. "Reduce $20 by 50 percent. What do we have?" Most students will be able to give the correct response of $10.
2. "Now reduce that $10 by 50 percent." A moment of hesitation is likely as students see that this new reduction takes them to $5.
3. "We have done two reductions of 50 percent. Shouldn't two reductions of 50 percent take us to $0?" Most students will be able to state that, no, we should not now be at zero; we took "50 percent off" two different amounts. The hope is that students will be able to recognize that the same phenomenon is at work as Pat's company loses money.

The teacher's goal is to lead students to state in their own words the phenomenon noted through the questions. One sixth-grade student commented, "When you chop off 10 percent of what you have, you have something smaller, and 10 percent of *it* will be smaller, too." That observation is praiseworthy. It indicates that the student has understood the essence of this mathematical phenomenon.

3. Busy Lawyers

Attorneys Barr and Writ are two highly competitive injury lawsuit lawyers. They are so competitive that twice a year, at the end of June and December, they compare their trial won-lost records, with the "winner" treating the other lawyer to dinner.

For the first half of a recent year, Barr's winning percent, the number of cases won divided by the total number of cases handled, was better than Writ's winning percent. For the second half of that year, Barr again had a better winning percent than Writ.

Is it possible that attorney Writ had a better winning percent than attorney Barr for that entire year? If so, how? If not, why not?

The Busy Lawyers

The classic Busy Lawyers problem is shown in **figure 1**. Please read it and answer the question at its end before proceeding.

The answer is yes. Writ could have had a better all-year winning percent than Barr in spite of Barr's having the better first half *and* the better second half.

Intuition loudly screams that Writ could not possibly have had a better yearlong winning percent than Barr. When first confronted with this problem, one seventh grader asserted that "Barr beat Writ twice. There is no way that two wins could be a 'lose.'" Barr beat Writ in the first half. Barr beat Writ in the second half; case closed.

Although these ideas are good opening arguments from students, the teacher should challenge them to push their thinking beyond the obvious. If students do not bring up the point on their own, the teacher should ask them whether the number of cases that each lawyer handled is relevant. (It is.) This question may spark some students to examine the situation more closely. Here are other questions that the teacher may want to pose if students are struggling with this issue.

1. Barr's yearlong winning percent will fall between what two other percents? (Answer: Barr's first-half and Barr's second-half winning percents.)
2. Using the same logic, where will Writ's yearlong winning percent lie?
3. Will Barr's and Writ's yearlong winning percents always be *exactly* halfway between their half-year winning percents? If not, what factor might determine to *which half-year percent* each lawyer's yearlong percent is closest?

The teacher's purpose here is to encourage students to rely on their intuition, which says no; each lawyer's yearlong winning percent may *not* lie exactly halfway between their half-year percents. This revelation may open up the possibility that Barr may have the better yearlong winning percent.

Table 1 describes a possible scenario in which Barr wins the individual halves of the year but Writ, nonetheless, becomes the yearlong champion. Note that the winning percent is given in baseball-statistics style, that is, as a decimal shown to the thousandths place.

An everyday example may help to clarify the issue for students:

> Jerry is given only one mathematics test during the first semester, and he earns a score of 100 percent. During the second semester, twelve tests are administered, and Jerry consistently earns a score of 80 percent on each one. Assuming that the teacher assigns equal weight to all thirteen tests, is it fair to take the first-semester score of 100 percent and average it with the consistent second-semester score of 80 percent for a yearly average of 90 percent?

Jerry might wish it so, but that approach is bad mathematics.

Table 1

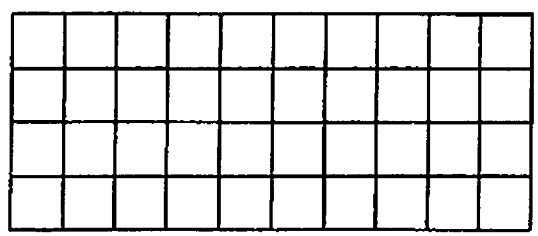
	Lawyer	Wins from Cases Tried	Winning Percent
First half of the year	Barr	60 out of 100	0.600
	Writ	29 out of 50	0.580
Second half of the year	Barr	40 out of 50	0.800
	Writ	79 out of 100	0.790
Entire year	Barr	100 out of 150	0.667
	Writ	108 out of 150	0.720

CHAPTER 5

Linking Fractions, Decimals, and Percents Using an Area Model

FIGURE 5.1. Opening activity to be completed prior to reading the case of Ron Castleman.

Shade 6 of the small squares in the rectangle shown below.



Using the diagram, explain how to determine each of the following:
(a) the percent of area that is shaded
(b) the decimal part of area that is shaded
(c) the fractional part of area that is shaded

THE CASE OF RON CASTLEMAN

Ron Castleman is an experienced teacher who demands much from his students. Before entering teaching, he was an engineer for a major corporation. Although he was considered successful at his work, he found himself unfulfilled in important ways and so, at mid-career, he returned to school to obtain a master's degree in teaching. Majoring in secondary mathematics was a natural choice, given his background and love of the *discipline. Interestingly, his natural ability at mathematics turned into a stumbling block rather than an asset once he began teaching middle school. Ron prided himself on not being the kind of teacher who presented meaningless algorithms. However, he found that his ways of thinking about concepts and procedures made very little contact with seventh graders' ways of thinking about them. He often became frustrated when his students didn't "get it" and blamed the students for not paying attention or not trying hard enough.*

It took Ron a long time to figure out why he was having such a difficult time connecting with his students. After several years of teaching, however, he learned not only to talk to his students but also to listen to them. This enabled him to begin to comprehend how they were understanding (or misunderstanding) mathematics. Over the years, one of the strategies that he found most useful for encouraging students to talk about mathematics was the use of diagrams. Ron believed that diagrams, although not a panacea by any means, provided a tool for reasoning. With them, he had seen mathematics come alive for his students.

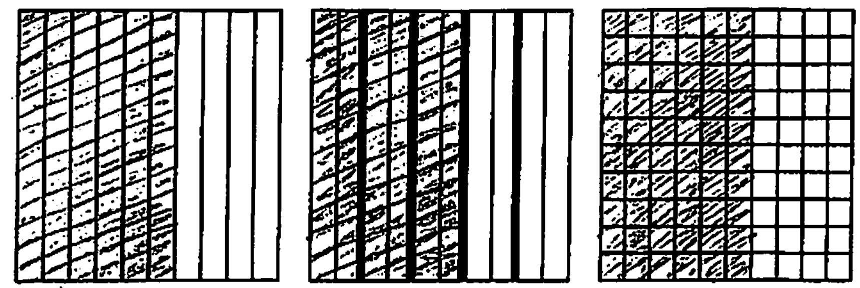
For the past several years, Ron has been teaching from a curriculum that makes extensive use of visual diagrams. He thinks that he and his students are beginning to communicate with each other in meaningful ways about important mathematical concepts, but he sometimes worries that procedures—those wonderfully quick and efficient ways of getting from point A to point B—may be getting lost in the process. His main struggle during the past year has been finding a balance between encouraging the development of conceptual understanding and the facile use of efficient procedures.

Ion Castleman Talks About His Classes

My seventh-grade students and I had been working with fractions and ecimals for several weeks. We began by learning the traditional conversion gorithms (e.g., $3/5$ = 3 divided by 5 = .6), but then moved on to investigaons of the meaning of fraction and decimal concepts. We did this by using ianipulatives and visual diagrams to focus on portions of a unit whole and lace value. For example, we used decimal squares divided into tenths and undredths (see Figure 5.2) to illustrate how the areas covered by $3/5$, $6/10$, .6, $/100$, and .60 are all the same.

Most recently, we began working with percents. The lion's share of the me was spent emphasizing the meaning of percent by using various manipu-tives and visual diagrams.

FIGURE 5.2. Decimal squares that illustrate that the areas covered by $3/5$, $6/10$, .6, $60/100$, and .60 are equivalent.



$6/10$ = .6 $\quad$ $6/10$ = .6 = $3/5$ $\quad$ $6/10$ = $60/100$ = .60

Both of my seventh-grade classes were nearing the end of the unit and I was anxious that they begin to pull together all of their work on rational numbers. On this particular day, I decided to have the students work with all three representations—fractions, decimals, and percents—at the same time. My goal was for the students to figure out the percent, decimal, and fraction representations of shaded portions of a series of rectangles. In particular, I wanted the students to use the visual diagrams to determine their numerical answers rather than relying on the traditional algorithms that we had learned at the beginning of the unit. I hoped this would help them develop conceptual understandings of each of these forms of representing fractional quantities and the relationships among them.

I planned on doing the same lesson with both my second-period and my sixth-period classes. This would give me my lunch period to reflect on the earlier class and make adjustments based on what worked and what didn't work. The two classes are amazingly similar in their makeup and in how they react to lessons, so I often find that this strategy works well. (Sometimes I try out the lesson on the sixth-period class first so that the second-period students are not always the guinea pigs!)

Second-Period Mathematics Class

Setup. At the beginning of the lesson, I passed out a set of three problems and asked the students to focus on the first one (shown in Figure 5.1 at the beginning of this chapter).

I expected the students to be challenged by this problem because it would be the first time that they would be working on a grid that was not 10×10.

This would add a layer of complexity because subregions constituting tenths or hundredths, for example, would not be automatically evident, but rather would have to be reasoned out based on students' understandings of fractional relationships.

As we discussed the problem, I indicated that I wanted them to actually use the diagram to figure out the answers to each part and that there was more than one way to do so. In my verbal instructions to the students, I clearly stated that I wanted them to be prepared to give explanations and/or to illustrate diagrammatically why their responses made sense. I told the students to work on the first problem for about 10 minutes with their partners. Then we would go over it as a class, with students demonstrating how they went about using the diagram to solve the problem.

Implementation. As students started to work in their pairs, I circulated around the room observing their approach to the problem. All students easily shaded the 6 squares. My first observation of student difficulty was their lack of success in figuring what percent the 6 squares were of the total diagram. Some had gleefully written 6%, failing to notice that the total number of squares was not 100, but 40. Most, however, did notice that the diagram was not the usual 10 × 10 grid, and that 6% would not be the correct answer. We had not learned an algorithm for determining the answer to "6 is x% of 40" and students appeared stumped as to how to proceed. I let them struggle for a short time, but became increasingly uncomfortable with their lack of progress. Several students, feeling frustrated by their inability to quickly determine the answer, began to press me for an algorithm to use to figure the correct percent. I hesitated. I did not want to provide the students with a method for finding the percent (e.g., $6/40 = x/100$); I wanted them to use the diagram, but I wasn't sure how to get them to do so. Many students appeared to have run into a brick wall and were becoming increasingly uneasy with the ambiguity.

After a quick assessment of the problem, I noticed that starting with part C (What fractional part of the area is shaded?) would perhaps be easier. As I visited pairs, I suggested that they begin by figuring the fraction first. My instincts were correct; most students successfully used the rectangle to figure that the 6 shaded squares would be represented by the fraction $6/40$, which could be reduced to $3/20$. What happened next, however, was surely not in my game plan, but I found myself incapable of stopping it. Students excitedly noticed that they could now do part B (What decimal part of the area is shaded?) by simply dividing 3 by 20 and coming up with the decimal answer of .15. And once they knew that the decimal representation was .15, they quickly turned to the "tri[illegible]ed-true" method of moving the decimal point two places to the right to co[illegible]ert from decimals to percents. At this point, they believed that they had completed the problem and waited for me to visit their group to verify that their calculations were correct. I, however, was not convinced that they understood the reasoning behind the conversions they had performed, nor the relationship between the fraction/decimal/percent and the shaded area of the rectangle. They didn't seem to be using the diagram, not even to check the reasonableness of their answers.

It was now 15 minutes into the period and we needed to move along. After stopping by each table and marking their answers right or wrong, I said that it was time to share answers with the entire class and called on a pair of students, Jena and Ray, to display their methods and answers. I had noticed that these two students had made a calculation error and so I planned to make an appeal to return to the diagram to check the reasonableness of their answers. Jena and Ray had correctly figured $3/20$ as the answer to the part C, but had divided inaccurately and come up with an answer to part B of .015. Their answer to Part A was 1.5%. After they had displayed their calculations and answers, I said, "Look at the diagram. Does it look like only 1.5% of the rectangle is shaded?" Before they could answer, however, several students at their seats began pointing to the decimal placement error in the students' division problem. I felt that I couldn't ignore this either, so I did a quick review of the procedure for long division with decimals. Once the correct answer of .15 had been calculated, Jena and Ray quickly changed their percent answer to 15% and went back to their seats. I pointed out to the class that 15% surely seemed like a more reasonable estimate of the percentage of the shaded area than did the first answer of 1.5%.

Although we had already spent 20 minutes on this one problem and had displayed the correct answers to all three parts, I was reluctant to leave it because I still had the uneasy feeling that we had spent too much time worrying about correct answers and procedures and not enough time actually using the diagram to reason with. I decided to take a risk and call to the overhead Sharice and Krystal, the only two students in the class who had not resorted to algorithms to solve the problem. Although they had not come up with the right answer, they had at least tried to reason using the diagram. Sharice and Krystal started by shading 4 squares in the first column and 2 squares in the second column and then stating that 6% of the squares was shaded. (I knew their answer was incorrect, but was not sure what to do to get them to reconsider it. It didn't matter; while I was thinking of what to say, they moved on, eager to show their work for the next part of the problem.) For part B, they said that they did not have a specific answer, but that they had an idea. Krystal pointed to the rectangle and said, "There are 10 columns all together, which means that each column is worth $1/10$. If one column was shaded, the answer would be .1 and if both columns were shaded the answer would be .2. S[illegible] answer falls halfway in between." When I asked them how they could calculate or write [illegible] of $\frac{1}{10}$ as a decimal, they shrugged their shoulders. At this point, I turned to the class asking if anyone at their seats knew how to write $\frac{1}{2}$ of $\frac{1}{10}$ as a decimal. A couple of students volunteered incorrect answers. Feeling pressed for time, I reminded the class of previous work that we had done in which the word *of* was translated into a multiplication sign. As I wrote on the blackboard, I explained, "$\frac{1}{2} \times \frac{1}{10}$ equals $\frac{1}{20}$, which could be written by the equivalent fraction of $\frac{5}{100}$ or the decimal of .05." I went on to explain that the decimal representation for the 6 shaded squares would be .15 ($\frac{1}{10} + \frac{5}{100}$).

Checking my watch, I saw that we were now nearly 30 minutes into our 45-minute class period. I decided that we had spent enough time on this problem and that the students should begin work on the remaining two problems.

STOP

Discuss second-period class.

Reflection on Second Period

Over lunch, I thought about what had transpired during the second hour. I also had the chance to review students' papers on the three problems. Most had indeed completed all three problems. However, the overwhelming majority of students had jumped directly to the use of algorithms, with little if any evidence of actually having paid attention to the diagrams. Moreover, the procedures they had used were sometimes the wrong ones; often, they had been inaccurately executed. To my surprise, I found myself being less upset by sloppy execution than by the fact that many students had obviously failed to check the reasonableness of their answers.

In thinking over the lesson, I decided that my objectives were reasonable and that I had chosen and set up good tasks. And the students were certainly capable of doing what I had asked, if they would only slow down and take the time to really think about what they were doing. I decided to try again with my sixth-period class, but this time not to allow the clock to govern our pace. Also, I vowed not to allow the students' anxiousness about how to proceed get to me. If I could only find a way to support them without telling them how to do it. . . .

Sixth-Period Mathematics Class

Setup. I set up the task in the exactly the same manner as I had with my second-period class, again stressing that they needed to use the diagram to figure out their answers. Once again, I promised to judge them by the quality of their explanations and visual reasoning processes in addition to whether they got the right answers.

Implementation. As students started to work in their pairs, I walked around the room observing their approach to the problem. As with the second-period class, my first observation of student difficulty was their lack of success in figuring what percent the 6 squares were of the total diagram. Once again, a few students had written 6%, failing to notice that the total number of squares was 40, not 100. Most, however, did notice that the diagram was not the usual 10×10 grid, and that 6% would not be the correct answer. This class had also not learned an algorithm for determining the answer to "6 is what % of 40" and, once again, the students appeared to be stumped regarding how to proceed. I held my breath and let them struggle.

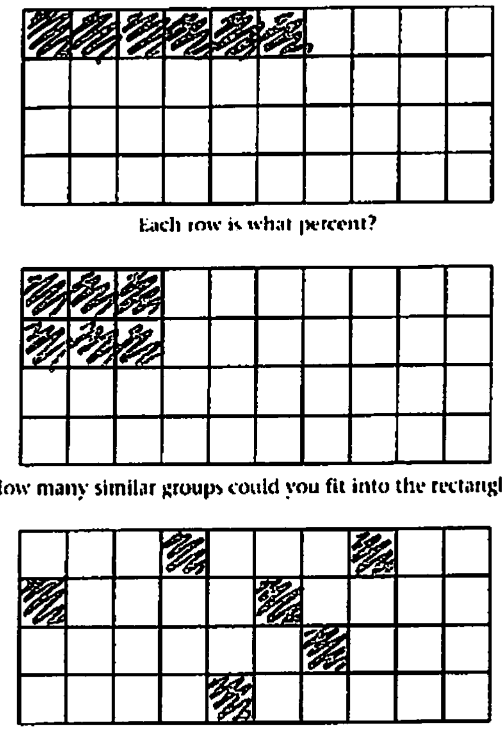
It didn't take long for several students to begin to complain that it was too difficult because they had not been taught the rule for figuring the correct percentage when the number of squares was not 100. In response, I tried to redirect their attention to the diagram. I suggested that they might want to look carefully at the rectangle, noticing both the total number of squares and the ways in which the squares were organized into columns and rows. "How might you use this information to help you figure the percentage?" I would ask. This sustained them for awhile. They stopped bugging me for a formula and I could hear the quiet buzz of conversation as students began discussing possibilities with their partners, pointing to their diagrams as they spoke.

Almost 10 minutes had passed and I started to get nervous. Most students still had not correctly figured the percent of shaded squares. I noticed that students were engaged with the task, however, and several appeared to be holding quite animated discussions with their partners. Remembering my pledge to myself, I decided to let them go a few minutes longer.

As I visited pairs, I looked carefully at the various ways the students were attacking the problem. I observed that those students who were making the most progress had noticed that each column represented $\frac{1}{10}$ of the rectangle and that 6 squares could be seen as "filling up" $1\frac{1}{2}$ columns. If a column was $\frac{1}{10}$ or 10%, then a "column and a half," they reasoned, would be 15%.

The students who were having the most difficulty were working with rectangles in which the shaded squares were not in columns, but rather were shaded horizontally in rows, in 2×3 rectangles, or in a scattered formation. This appeared to lead to difficulties in seeing the rectangle as divided into tenths. I tried to assist these students in finding other ways to figure the percentage by asking questions that would allow them to build on the particular configuration that they had shaded (see Figure 5.3).

FIGURE 5.3. Teacher questions that built on different shadings of six blocks (Stein & Smith, 1998).



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In each instance, I would say, "Think about that. How might it help you figure out the percent?" Then I would leave them on their own to work on it a bit longer.

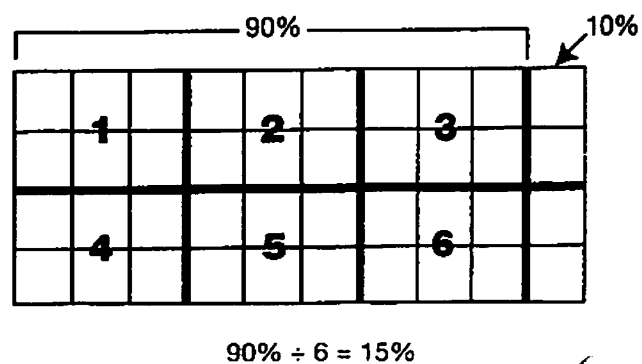
We were now about 20 minutes into the period and I noticed that most students had at least attempted all three parts of the problem. I decided that it was time to share strategies and rationales with the entire class. I called Jaleesa and Rachel to the overhead because they had approached Part A with a strategy that few others had used, one that I thought showed excellent thinking and reasoning. I asked them to explain their reasoning for Part A. Jaleesa wrote "6 × 2.5% = 15%" on the overhead transparency and then both girls began to return to the [illegible]ts. I quickly intervened, asking them to return to the overhead

projector and to provide a rationale for their method and their answer. Rachel explained that since there were 40 squares in the diagram and the whole diagram needed to represent 100%, each small square would have to equal 2½%. Since there were 6 shaded squares, they multiplied 2.5 times 6 to find out what percentage was shaded. I asked if other students had questions to ask of the girls about their solution method.

After Jaleesa and Rachel further clarified their method in response to questions from students at their seats, I thought that most of the students understood their approach. I took the opportunity created by this feeling of "we are all on the same page" to connect this pairs' reasoning strategies to an important mathematical idea. I noted that the girls began their explanation by stating that the rectangle equaled 100%. Michael asked, "How could something that wasn't subdivided into 100 equal 100%?" Derrick added, "Are you saying that 100% = 1? I thought that 100% = 100!" As students offered their views on these questions, we zeroed in on the important idea that 100% = 1 = 1.00. With respect to the current problem, we noted that 100% could be used to designate the whole rectangle, regardless of the number of parts into which the whole was subdivided.

Although we were now 30 minutes into the period, as a class, we had gone over only one strategy for one part of the problem. I decided to stick with the problem a bit longer and called a second pair of students to come to the overhead to display another way of figuring the percentage of shaded squares. Omar and Marcus had shaded 6 squares in the upper-left-hand corner of the rectangle and proceeded to figure the percent as shown in Figure 5.4. They explained their strategy and why it made sense while students at their seats listened and then asked a few questions.

FIGURE 5.4. The diagram displayed by Omar and Marcus.



Moving on to part B, I decided to call to the overhead Tim and Daniel. They had been working quite diligently, but had not yet reached an answer. Daniel began by pointing to the rectangle and saying, "There are 10 columns all together, which means that each column is worth $\frac{1}{10}$. If one column was shaded the answer would be .1, and if two columns were shaded the answer would be .2. So, the answer falls halfway in between." When I asked them how they could figure out what $\frac{1}{2}$ of $\frac{1}{10}$ was, they were not sure. At this point, I provided them with 2 decimal squares (see Figure 5.2) and asked them if the squares could help them to figure out what $\frac{1}{2}$ of $\frac{1}{10}$ would be. Using the hundredths square, Tim reasoned that $\frac{1}{10}$ was 1 column of 10 small squares (10 hundredths). One-half of $\frac{1}{10}$, he noted, would be $\frac{1}{2}$ of a column, or 5 small squares (5 hundredths). Returning to the original 40-square diagram, Daniel added that one column was $\frac{1}{10}$ or 10 hundredths (even though it had 4 squares!) and that $\frac{1}{2}$ of a column would still be half of that (that is, $\frac{1}{2}$ of 10 hundredths) or 5 hundredths (even though it had 2 squares!). The boys concluded that their answer needed to be 10 hundredths (.10) plus 5 hundredths (.05) for a total of 15 hundredths (.15).

We had run out of time! Although more time was spent on this first problem than I had intended, I thought it had been necessary to set the students on the right track for the remaining problems. I told the students to complete the remaining two problems for homework and to write explanations about how they used the rectangle to figure out their answers.

STOP

Discuss Ron's reflections and the sixth-period class.

DISCUSSION QUESTIONS

Following the Second-Period Class

1. What are some mathematical issues Ron was concerned with during the lesson? Why are these important issues? What nonmathematical issues did Ron seem to be concerned about?
2. How would you describe the thinking Ron was asking students to engage in when he set up the task? Did Ron's goals change after the students began working on the task? Were Ron's goals accomplished?

Following the Sixth-Period Class

3. How[illegible]ld you describe the thinking Ron was asking students to engage in when he set up the task for his sixth-period class?

   a. Did Ron seem to have the same initial goals for both classes?
   b. Did the students in the sixth-period class engage with the task in the same way as those in the second-period class? What was learned in each class?
4. What classroom-based factors might have contributed to the different kinds of student engagement in the two classes?
   a. What did Ron do differently in the two classes that might have supported (or not supported) students' engaging in the task as he intended? What, if anything, did he do that was the same?
   b. What did the students do in each class that might have influenced their own learning and engagement with the task?

TEACHING NOTES

Ron Castleman is a teacher who knows his subject area well and who sincerely wants his students to know and understand mathematics in a deep way. He is also a teacher who learns from his experiences. He has changed his teaching practices to incorporate the use of visual diagrams and to include more student discussion. He also has established a "reflection routine" in which he uses his experiences in his second-period seventh-grade class to inform his approach in his sixth-period seventh-grade class.

This year, Ron has been struggling with how to balance the teaching of efficient procedures and the encouragement of students' development of conceptual understanding—a dilemma not unfamiliar to many teachers of mathematics. In this case, Ron encounters another, related dilemma. With the visually oriented curriculum, he has been able to introduce his students to increasingly complex and interesting problems. He is discovering, however, that the students sometimes prefer simpler, more straightforward tasks and can react anxiously to the uncertainty associated with not knowing immediately how to tackle a more complex, less structured task. Ron clearly wants his students to know *both* procedures and concepts, but he is having some difficulty getting them to discard the procedures and focus on meaning in the pair of lessons that are featured in this dual case. He is more successful in the second class (sixth period) than the first (second period). In order to understand this case, participants will have to recognize this and be able to identify the factors responsible for his greater success with the sixth-period class.

Cognitive Levels

The same task is used in both classes and is set up in essentially the same manner. In both classes, we consider the task—as set up—to b[illegible] he *doing-*

mathematics level. If the task is solved in the manner requested (i.e., by using the diagram and not using the traditional conversion procedures), it demands complex thinking and reasoning and a considerable amount of cognitive effort. The complexity of the task is largely derived from the fact that the grid is not a 10 × 10 square, and hence, students must construct novel ways of configuring the squares in order to determine the correct percentages and decimals (as opposed to relying on a grid that is conveniently laid out in tenths). When the visual diagram is used to solve the problem in this way, individuals are challenged to apply and use their understandings of fraction, decimal, and percent concepts in novel ways. When announcing the task to his students, Ron maintained these high levels of demand. He stressed that students should use the diagram to reason out their answers, noting that there was more than one way to do so. He also warned students that they should be prepared to give explanations and/or to illustrate diagrammatically why their responses made sense.

The ways in which students actually went about working on the task (the task-implementation phase) differed, however, in the two classes. We consider the implementation of the task during the second-period class to be at the level of *procedures without connections*. The vast majority of students performed algorithmic procedures that departed completely from the diagram. After starting with part C (finding the fraction), students found that they were able to use traditional conversion procedures that had previously been taught in order to find the answers to the other two parts (the decimal and percent representations). Moreover, the students did not even return to the diagram to check the reasonableness of their answers (e.g., two students got 1.5% for their answer and failed to recognize its absurdity).

During the sixth-period class, on the other hand, students did not start by finding the fraction, but rather did the problems in the suggested order; hence, they had to struggle with finding what percent of 40 the 6 shaded squares would be. Students were forced to use the diagram to reason out the answer, because they knew of no rules to find the answer. They used a variety of strategies to reason their way through the task, most of which required that they apply and use their understandings of percent, decimal, and fractions. We consider the implementation of the task during the sixth period to be at the level of *doing mathematics*.

Factors

Superficially, the implementation portions of the two lessons were conducted in si[illegible]lar ways: The students worked in pairs with careful teacher

monitoring, the teacher made decisions based on what he noticed about student work, and the teacher called students to the front of the room to show their work. At a deeper level, however, Ron's actions in the two classes differed. Despite being faced with similar pressures as the two lessons unfolded, Ron handled the difficulties in very different ways in the two classes. These differences constitute the factors that enabled one lesson to stay at a high level while the other lesson declined into a procedural implementation. *These are the ideas toward which the case discussion should be steered.*

Maintaining problem complexity; assisting through scaffolding. During the second-period class, Ron inadvertently simplified the problem by suggesting that students start with finding the fraction; he did not repeat this mistake in the later class, however. When the sixth-period students pressed him for help, he instead urged them to look more closely at the visual organization of the diagram and to think about how that could be mapped onto mathematical ideas that could help them figure out the percentage, thus keeping the search for meaning and the connection to concepts foregrounded. He thus supported their thinking in a way that maintained the integrity of the task, rather than changing the task into a simpler procedure. At other points during the second-period class, Ron was also led into procedural spaces (e.g., he found himself demonstrating the long-division and multiplication-of-fractions algorithms). During the sixth-period class when Ron was faced with the same request for how to determine ½ of ⅒, Ron did not allow himself to be drawn into a procedural explanation, but rather supplied students with the decimal-squares representations, which allowed them, in turn, to reason the answer out on their own.

Another way in which Ron supported students' thinking while maintaining task complexity was the manner in which he built on whatever configuration of squares the students had shaded. Depending on which particular set of squares was shaded, Ron asked a different set of questions that could subtly steer the students toward the recognition of patterns that would be helpful in reasoning through the task.

Sustained press for justification, explanations, and meaning. During the morning class, Ron appeared to be more concerned with relieving students' anxieties than with having them struggle to construct meaning. There were subtle cues given to students that arriving at the correct answer was paramount (e.g., he stopped by each desk, marking their answers right or wrong) and that explanations were not really expected (students who were asked to justify their answer of .015 never had a chance to speak). In [illegible] afternoon class, however, Ron held students accountable for explanations and justifications throughout the implementation phase. For example, he did not allow the students who wrote "$6 \times 2.5 = 15\%$" to sit down until they had explained how they arrived at their answer and entertained questions from the class. Similarly, he demanded that the second pair of students reason their way through figuring ½ of ⅒ in a meaningful way.

Building on student knowledge and thinking. In both classes, Ron tried to build on students' prior knowledge, but he drew on different kinds of knowledge in the two classes. In the second-period class, Ron resorted to what they had learned about procedures, for example, how to perform long division or multiply fractions. In the sixth-period class, on the other hand, Ron tried to get students to use more conceptual forms of knowledge. For example, when the final pair of students had difficulty determining ½ of ⅒, he supplied them with the decimal-squares representations in an attempt to make accessible the hundredths place value and to illustrate (visually) the relationship between hundredths and tenths. Such translations between hundredths and tenths using the decimal-squares representations had previously been done in class and it was this knowledge that Ron chose to build on.

Teacher makes conceptual connections. Most of the connections that Ron made in the morning class were to procedures. In the afternoon class, the most directive segment was Ron's guiding the students toward the recognition that $100\% = 1 = 1.0$. Being able to identify the unit whole for percentages, fractions, and decimals is an important foundational concept for this kind of work and Ron took advantage of a "teachable moment" to point this out.

Modeling high-level thinking. In the second-period class, there was no opportunity for the students to see and hear what a "high-level solution process" to the problem would look like. In the sixth-period class, on the other hand, Ron was careful to select some students to present well-thought-out solutions to the entire class. Two of the whole-class presentations represented two viable methods of figuring the percentage; Ron made sure that the students explained their thinking processes as well as simply illustrating their answers. The final whole-class presentation was not completely worked out, but this provided an excellent opportunity for students to witness others "thinking out loud" about how to figure what ½ of ⅒ was.

Provision of enough time. Ron allowed himself to be pushed by the clock in the second-period class. Although students completed more problems, it is

unclear that they understood as much as did the students in the sixth-period class. During the sixth-period class, Ron was aware of the passage of time but he did not allow it to close off productive student engagement. Students had enough time to work their way through the task in a meaningful way and to view how others had done so.

Additional Layers of Interpretation

An additional issue that this dual case raises for discussion is the manner in which students should be provided with opportunities to learn both deep-seated mathematical ideas and concepts *and* algorithmic procedures. This was a goal of Ron Castleman's, a goal with which he still appeared to be struggling. Indeed, the integration of procedural and conceptual knowledge or low-level and high-level skills is a concern and dilemma for many teachers of mathematics.

Ron taught the procedural algorithms first and then moved on to more conceptually based work. Participants may suggest that perhaps he should try the reverse order, beginning instead with the visually based work (indeed, this is the suggestion of the NCTM). By doing so, he will first be laying a conceptual foundation, without the interference of algorithms. After the students appear to have a good grasp of the meanings of percents, decimals, and fractions, they could be provided with the conventional conversion algorithms, with special attention to places where connections to underlying concepts could be made.

Another issue that could be discussed related to this case is the usefulness of teacher reflection for the improvement of instructional practice. Ron provides a good example of a teacher who reflects both on his own actions and on what students appear to be learning.

POSSIBLE SOLUTION STRATEGIES

Our discussion of possible solution strategies is presented in three sections based on three broad approaches to solving this problem. First, we discuss algorithmic solutions; we then move onto the discussion of solution strategies that are based on reasoning from the visual diagram. Finally, we end with a solution strategy that is based on extending the diagram.

Algorithms/Procedures

Algorithmic solutions are those based on learned procedures; students often implement these without understanding why or what th[illegible] are doing.

Percent. Use a proportion to determine the percent.

$\frac{6}{40} = \frac{N}{100}$	*Since percent is based on 100.*
$40 \times N = 6 \times 100$	*Product of the means = the product of the extremes*
$40 \times N = 600$	
$N = \frac{600}{40}$	
$N = 15$	*The 6 shaded blocks represent 15%.*

Decimal. Consider that 6 parts out of 40 are shaded and divide the numerator by the denominator following the appropriate procedures for decimal division. The 6 shaded blocks represent .15.

Fraction. The fraction is the number of parts shaded compared with the total number of parts in the whole. The 6 shaded blocks represent $\frac{6}{40}$ or $\frac{3}{20}$.

Reasoning from the Visual Diagram

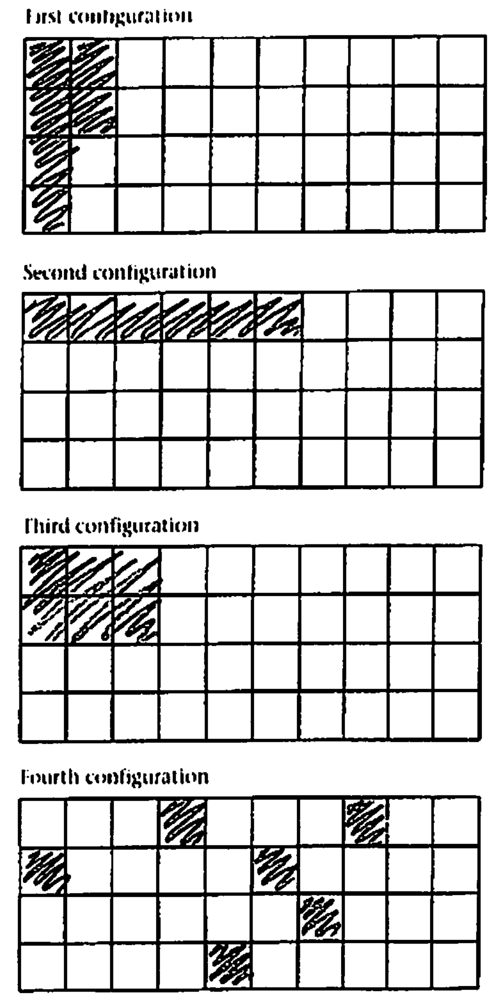
Students may have shaded in the 6 squares in a variety of ways. Four different configurations of shading that we've often seen are illustrated in Figure 5.5. The way in which students might reason about each of these configurations to determine their percent and decimal answers is described below. We assume their understanding of fractions as part/whole relationships would make determining the fraction answer of $\frac{6}{40}$ or $\frac{3}{20}$ trivial; therefore, part C is not explained.

First configuration. Students may have begun by shading in 4 squares in the first column and 2 squares in the second column.

1. Percent. The entire rectangle represents 100%. Since there are 10 columns in the rectangle, the one column of 4 shaded squares will be $\frac{1}{10}$ of the rectangle, or 10%. The second column has only 2 squares shaded, or ½ of the column. If the whole column is 10%, then half the column is 5%. Thus, the 6 shaded blocks that make up 1½ columns equal 10% plus 5% or 15%.

2. Decimal. The entire rectangle represents one whole. Since there are 10 columns in the rectangle, the one column of 4 shaded squares will be .1 of the rectangle or .10. The second column has only 2 squares shaded or half the column. Half of .1 or half of .10 is .05. (or the student might reason: ½ of $\frac{1}{10}$ is $\frac{1}{20}$ or $\frac{5}{100}$). Thus, the 6 shaded blocks that make up 1½ columns equal .1 plus .05, which equals .15.

FIGURE 5.5. Four configurations of shading associated with different solution strategies.



Second configuration. Students may have begun by shading in 6 squares horizontally across the first row.

1. Percent. The entire rectangle represents 100%. Since there are 4 rows in the rectangle, one row is ¼ of the rectangle, or 25%. Since there are 10 blocks in a row, one block must represent 2.5%; 2 blocks would represent 5%; 4 blocks would represent 10%; and 6 blocks would represent 15%.

2. Decimal. The entire rectangle represents one whole. Since there are 4 rows in the rectangle, one row is ¼ of the rectangle, or .25. Since there are 10 blocks in a row, 6 blocks represent .6 of the row or .6 × .25 of the whole; .6 × .25 = .15.

Third configuration. Students may have begun by shading in 3 squares horizontally in each of the first two rows. We call this the "postage stamp" configuration.

1. Percent. The entire rectangle represents 100%. I can partition the rectangle into six groups of 2 × 3 sections, leaving one column on the end. The column on the end is 1/10 or 10% because there are 10 columns in the rectangle. That means the six 2 × 3 sections take up the remaining 90% of the rectangle. Ninety percent divided by 6 is 15%. So, one section of 6 blocks would represent 15%.

2. Decimal. The entire rectangle represents one whole. I can partition the rectangle into six groups of 2 × 3 sections, leaving one column on the end. The column on the end is 1/10 or .1 because there are 10 columns in the rectangle. That means the six 2 × 3 sections take up the remaining .9 of the rectangle, and .9 divided by 6 is .15. So, one section of 6 blocks would represent .15.

Fourth configuration. Students may have begun with a random shading of the 6 squares throughout the diagram.

1. Percent. The entire rectangle represents 100%. Since there are 40 blocks, 80% can be distributed across the 40 blocks by giving 2% to each block. That leaves 20%. Since 20 is half of 40, the 20% can be distributed across all the blocks by giving each block another .5%. Thus, each block would represent a total of 2.5% (2% +.5%). Two blocks would represent 5%, 4 blocks would represent 10%, and 6 blocks would represent 15% (or 6 times 2.5% = 15%).

2. Decimal. The entire rectangle represents one whole. Each block is 1/40, but I don't know what the decimal equivalent of that is. If I think about putting the blocks together into pairs, then there would be 20 pairs and each pair of blocks would be 1/20 or .05 because there are 20 nickels in $1.00. So, the 6 blocks are 3 pairs or 3 nickels, which would be .15.

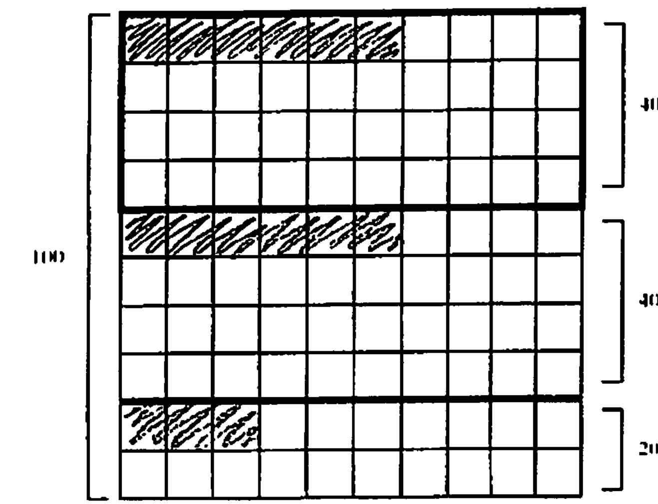
Reasoning by Extending the Diagram

In this solution strategy, students begin by extending the diagram to 10 × 10 (see Figure 5.6).

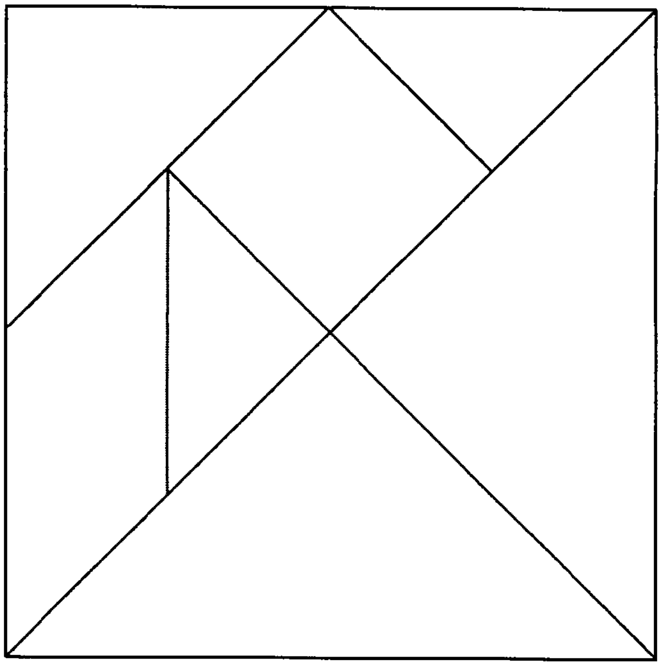
Percent. If there were 100 squares, then the number of shaded squares would equal the percent. There are 40 squares to begin with and 6 are shaded. If another 40 squares are added, then 6 more would be shaded. To get 100 squares you need to add 20 more. Since 20 squares is half the original 40 squares, then only half as many squares would be shaded. So 3 of the 20 squares would be shaded. Altogether then, 6 + 6 + 3 = 15 of the 100 squares would be shaded. Fifteen out of 100 is 15%.

Decimal. If the whole is made up of 100 squares, then each shaded square represents .01. Since there are 15 squares shaded, this is equal to .15.

FIGURE 5.6. Extending the diagram in Ron Castleman's task to a 10 × 10.

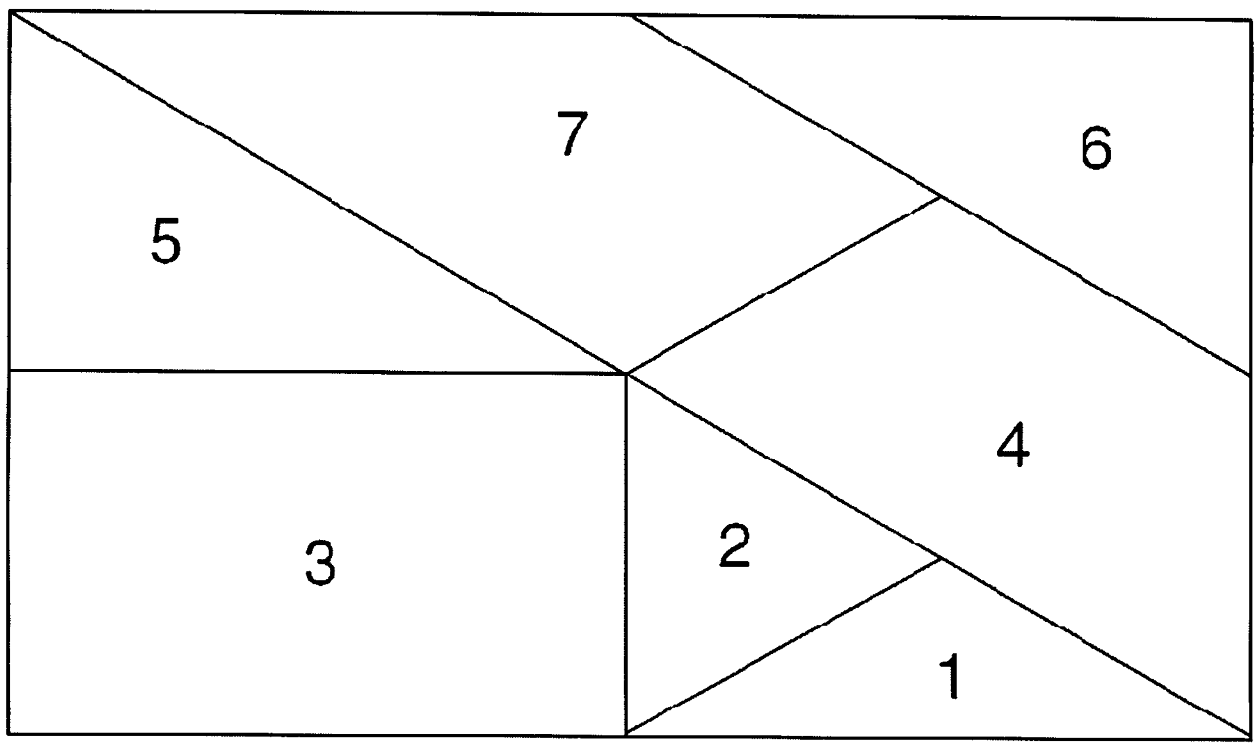


Activity 3 – Tangram and Mosaic Puzzle Calculations



If is the unit whole, determine:

Shape	Fraction	Decimal	Percent
Large Triangle			
Large Triangle			
Medium Triangle			
Small Triangle			
Small Triangle			
Square			
Parallelogram			
Total			



If is the unit whole, determine:

Shape	Fraction	Decimal	Percent
1			
2			
3			
4			
5			
6			
7			
Total			

Activity 4
Smartie Lab

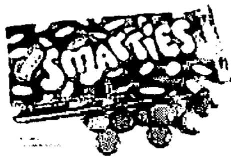
Materials

1 big box of smarties/ 2-3 students
Worksheet (Smartie Lab Worksheet)
Pencil
Calculators

Procedure

1. Choose a group of 2 or 3 students and obtain 1 box of Smarties and the Smartie Lab worksheet.
2. Count the total number of Smarties in your box. (DO NOT EAT)
3. Sort your Smarties by colour.
4. Record what fraction of your Smarties were each colour.
5. Convert each fraction to a decimal and a percent.
6. Compare results with your classmates.
7. Determine how many Smarties each person in your group must get in order for the Smarties to be shared fairly. Write this number as a fraction, decimal, and a percent.
8. Eat your Smarties.

Smartie **Lab**



Total Number of Smarties ____________

Color ____________________

Percent ____________________

Decimal ____________________

Fraction ____________________

Color ____________________

Percent ____________________

Decimal ____________________

Fraction ____________________

Color ____________________

Percent ____________________

Decimal ____________________

Fraction ____________________

Color ____________________

Percent ____________________

Decimal ____________________

Fraction ____________________

Color ____________________

Percent ____________________

Decimal ____________________

Fraction ____________________

Color ____________________

Percent ____________________

Decimal ____________________

Fraction ____________________

Activity 5 – Geoboard Areas

Each student will need:

- an 11 x 11 pin Geoboard
- 2 copies (or 1 double-sided sheet) of the "Geoboard Record Sheet."

Instructions:

Use an elastic band to enclose:

1. $\frac{1}{10}$ of the Geoboard
   a) Show the area you enclosed on your Geoboard tracking sheet.
   b) Write this amount as a decimal and a percent.

2. 68% of the Geoboard
   a) Show the area you enclosed on your Geoboard tracking sheet.
   b) Write this amount as a fraction and a deceimal.

3. 0.5 of the Geoboard
   a) Show the area you enclosed on your Geoboard tracking sheet.
   b) Write this amount as a fraction and a percent.

4. $\frac{1}{4}$ of the Geoboard
   a) Show the area you enclosed on your Geoboard tracking sheet.
   b) Write this amount as a decimal and a percent.

5. 32% of the Geoboard
   a) Show the area you enclosed on your Geoboard tracking sheet.
   b) Write this amount as a fraction and a deceimal.

6. 0.13 of the Geoboard
   a) Show the area you enclosed on your Geoboard tracking sheet.
   b) Write this amount as a fraction and a percent.

7. Enclose a shape on the Geoboard in the shape of one of your initials.
   a) Show the area you enclosed on your Geoboard tracking sheet.
   b) Write this amount as a fraction, a decimal, and a percent.

8. Enclose a shape on the Geoboard of your own design.
   a) Show the area you enclosed on your Geoboard tracking sheet.
   b) Write this amount as a fraction, a decimal, and a percent.

Geoboard Record Sheet

Name___________________________

Sports Math

For each teammate, including yourself, record the number of shots attempted and the number of shots made. Represent each stat as a fraction, decimal, and percent.

Teammate's Name	Number of Tries	Number of Shots Made	Fraction	Decimal	Percent

1. Which teammate had the highest percentage of shots made?

2. Which teammate had the lowest percentage of shots made?

3. Do you think the number of tries has a big effect on the percentage? Explain your answer.

Percents Journal

You will need a sheet of paper and coloured pencils.

Divide the paper into these 4 sections.

- ✓ 1 blue section that is ½ of the page.
- ✓ 1 red section that is 10% of the page.
- ✓ 1 yellow section that is 25% of the page.
- ✓ 1 green section to fill the remaining space.

Explain how you did this and determine what percent of the page is the green section? How do you know?

Activity 6 – Repeating Decimals Exploration

1. Use your calculator to convert each of the following fractions to a decimal.

   a) $\frac{1}{9}$ b) $\frac{2}{9}$ c) $\frac{3}{9}$

   d) Do you notice a pattern?

2. Write each of the decimals from question 1 using bar notation.

   a) ________ b) ________ c) ________

3. Use the pattern from question 1 to convert each of the following decimals to a fraction. Verify your solution using your calculator.

   a) 0.5555... b) $0.\bar{6}$ c) 0.77777... d) $0.\bar{8}$

4. Use your calculator to convert each of the following fractions to a decimal.

   a) $\frac{1}{99}$ b) $\frac{22}{99}$ c) $\frac{83}{99}$

   d) Do you notice a pattern?

5. Write each of the decimals from question 4 using bar notation.

   a) ________ b) ________ c) ________

6. Use the pattern you noticed in question 4 to convert each of the following decimals to a fraction. Verify your solution using your calculator.

   a) 0.4343... b) $0.\overline{21}$ c) 0.0505... d) $0.\overline{87}$

7. Use the pattern you noticed in question 1 and question 4 to convert each of the following decimals to fractions. Verify your solution using your calculator.
   a) 0.987987987...
   b) $0.\overline{341}$
   c) 0.006006006...
   d) $0.\overline{707}$

8. Write a fraction that you think will have one repeating digit. Convert the fraction to a decimal to verify.

9. Write a fraction that you think will have two repeating digits. Convert the fraction to a decimal to verify.

10. Write a fraction that you think will have three repeating digits. Convert the fraction to a decimal to verify.

Name: ______________________ Date: ______________________

BLM 4-5

Section 4.2 Extra Practice

For #1 to #3:

a) *Rewrite the fraction as a division expression.*
b) *Use a calculator to convert the fraction to a decimal number.*
c) *Round the decimal to the nearest thousandth.*
d) *Multiply the decimal by 100 to convert to a percent.*

	Division Expression	**Decimal**	**Round to Nearest Thousandth**	**Percent**
Example: $\frac{13}{18}$	**a)** 13 ÷ 18	**b)** 0.72222222	**c)** 0.722	**d)** 0.722 × 100% = 72.2%
1. $\frac{11}{12}$				
2. $\frac{15}{42}$				
3. $\frac{324}{365}$				

For #4 to #7, use a calculator to change each fraction to a repeating decimal. Show the answer in two ways.

Examples: $\frac{1}{3} = 0.33333\ldots = 0.\overline{3}$ $\frac{3}{11} = 0.272727\ldots = 0.\overline{27}$

4. $\frac{2}{3}$ ______________________ **5.** $\frac{5}{9}$ ______________________

6. $\frac{1}{13}$ ______________________ **7.** $\frac{3}{7}$ ______________________

For #8 to #11, calculate the following percents of each number:
a) *50%* ***b)*** *10%* ***c)*** *60%* ***d)*** *30%*

Note: ***c)*** *and* ***d)*** *are a combination of* ***a)*** *and* ***b)***.

	a) 50%	**b) 10%**	**c) 60%**	**d) 30%**
Example: 60	30	6	30 + 6 = 36	30 − 12 = 18 or 6 × 3 = 18
8. 40				
9. 90				
10. 200				
11. 150				

Name: ______________________________ Date: ______________________

BLM 4–5
(continued)

For #12 to #14, follow the steps to estimate each fraction as a percent.

12. $\frac{46}{80}$

a) 50% of 80 = ____

b) 10% of 80 = ____

c) a) ____ + b) ____ = ____

d) Therefore, $\frac{46}{80}$ is between ____% and ____% but closer to ____%.

13. $\frac{13}{30}$

a) 50% of 30 = ____

b) 10% of 30 = ____

c) a) ____ – b) ____ = ____

d) Therefore, $\frac{13}{30}$ is between ____% and ____% but closer to ____%.

14. $\frac{27}{40}$

a) 50% of 40 = ____

b) 10% of 40 = ____

c) a) ____ + b) ____ + b) ____ = ____

d) Therefore, $\frac{27}{40}$ is between ____% and ____% but closer to ____%.

15. Make up three fractions of your own and estimate each one as a percent as you did in #12 to #14.

Activity 7

Math 7 Name: ______________________

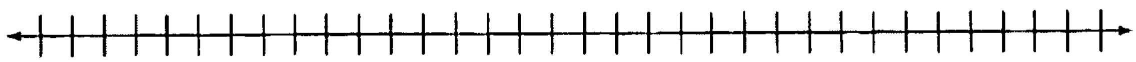
Comparing and Ordering Numbers

1. Complete the following:
   a. Name a fraction between ½ and 1. Convert this number to a decimal.

   b. Label the number line below to show these fractions.

   c. Name a decimal between ½ and 1 (must not be equivalent to the fraction in part a). Convert this number to a fraction.

   d. Place this decimal on the number line.

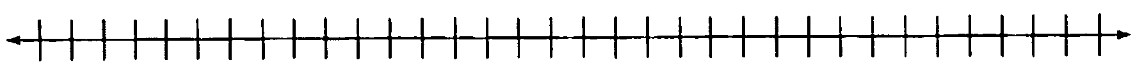


2. Complete the following:
   a. Name a fraction between ¼ and ¾, other than ½. Convert this number to a decimal.

   b. Label the number line below to show these fractions.

   c. Name a decimal between ¼ and ¾ (must not be 0.5). Convert this number to a fraction.

   d. Place this decimal on the number line.

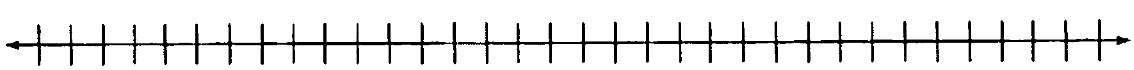


3. Complete the following:
   a. Name a fraction between ¼ and ½ whose denominator is 10. Convert this number to a decimal.

   b. Label the number line below to show these fractions.

   c. Name a decimal between ¼ and ½ (must not be equivalent to the fraction in part a). Convert this number to a fraction.

   d. Place this decimal on the number line.



4. Complete the following:

   a. Name a fraction between $\frac{7}{8}$ and 1. Convert this number to a decimal.

   b. Label the number line below to show these fractions.

   c. Name a decimal between $\frac{7}{8}$ and 1 (must not be equivalent to the fraction in part a). Convert this number to a fraction.

   d. Place this decimal on the number line.

5. Complete the following:

   a. Name a fraction between 0 and $\frac{1}{10}$. Convert this number to a decimal.

   b. Label the number line below to show these fractions.

   c. Name a decimal between 0 and $\frac{1}{10}$ (must not be equivalent to the fraction in part a). Convert this number to a fraction.

   d. Place this decimal on the number line.

6. Sort the fractions into the following 3 groups:

$\frac{4}{7}, \frac{1}{7}, \frac{8}{9}, \frac{4}{9}, \frac{3}{7}, \frac{5}{12}, \frac{2}{12}, \frac{9}{11}, \frac{6}{11}, \frac{4}{5}, \frac{3}{8}, \frac{13}{15}, \frac{1}{3}, \frac{5}{8}, \frac{7}{8}, \frac{4}{15}, \frac{7}{15}, \frac{2}{5}, \frac{2}{6}, \frac{6}{7}, \frac{5}{7}$

<u>Close to 0</u>   <u>Close to ½</u>   <u>Close to 1</u>

7. Order each set of numbers from least to greatest.

   a) $\frac{2}{10}, \frac{1}{7}, \frac{5}{8}, \frac{7}{10}, \frac{4}{7}$

   b) $\frac{7}{13}, \frac{6}{10}, \frac{1}{3}, \frac{1}{4}, \frac{55}{100}$

   c) $\frac{1}{3}, \frac{2}{9}, \frac{1}{2}, \frac{5}{6}, \frac{2}{3}$

8. Complete the following:
   a. Name a fraction between 2 ½ and 4. Convert this number to a decimal.

   b. Label the number line below to show these fractions.

   c. Name a decimal between 2 ½ and 4 (must not be equivalent to the fraction in part a). Convert this number to a fraction.

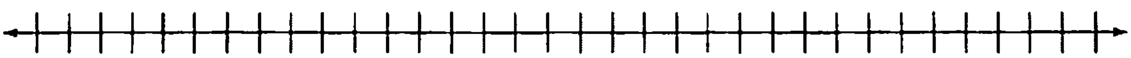
   d. Place this decimal on the number line.

9. Complete the following:
   a. Name a fraction between 1 ¼ and 2 ¾, other than 1 ½ or 2 ½ . Convert this number to a decimal.

   b. Label the number line below to show these fractions.

   c. Name a decimal between 1 ¼ and 2 ¾ (must not be 1.5 or 2.5). Convert this number to a fraction.

   d. Place this decimal on the number line.



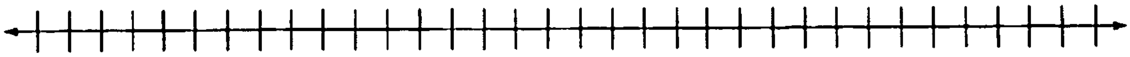
10. Complete the following:

a. Name a fraction between $4\frac{1}{4}$ and $5\frac{1}{2}$ whose denominator is 10. Convert this number to a decimal.

b. Label the number line below to show these fractions.

c. Name a decimal between $4\frac{1}{4}$ and $5\frac{1}{2}$ (must not be equivalent to the fraction in part a). Convert this number to a fraction.

d. Place this decimal on the number line.



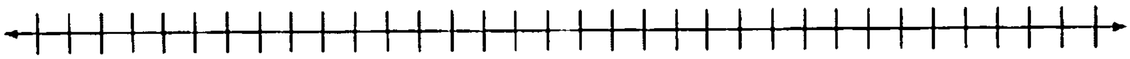
11. Complete the following:

a. Name a fraction between $2\frac{7}{8}$ and $3\frac{3}{8}$ that has a denominator other than 8. Convert this number to a decimal.

b. Label the number line below to show these fractions.

c. Name a decimal between $2\frac{7}{8}$ and $3\frac{3}{8}$ (must not be equivalent to the fraction in part a). Convert this number to a fraction.

d. Place this decimal on the number line.



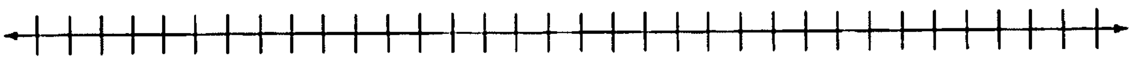
12. Complete the following:

a. Name a fraction between 3 and $3\frac{1}{10}$. Convert this number to a decimal.

b. Label the number line below to show these fractions.

c. Name a decimal between 3 and $3\frac{1}{10}$ (must not be equivalent to the fraction in part a). Convert this number to a fraction.

d. Place this decimal on the number line.



13. Order each set of numbers from greatest to least.

a) $4\frac{1}{7}$, 4.62, 5, $\frac{30}{8}$, $4\frac{3}{5}$, 5.1, $4.\overline{1}$, 4, $\frac{13}{3}$

b) $\frac{2}{3}$, $\frac{15}{7}$, 2, 0, $\frac{4}{3}$, 1.15, 2.64, $1.\overline{4}$, $2\frac{2}{5}$

Activity 8

Group 1: Using Percents to Calculate Discounts

Important things to know about Discounts:

Example Questions

1. Clare found a sale on a leather jacket. The regular price is \$288.60 but the sale is 35% off. How much will the jacket cost Clare?

2. Bobby wants to buy a new stereo that is 10% off. The stereo's regular price is \$48.50. How much will the stereo cost?

3. Hockey Sticks are regularly priced at \$29.99. They are marked 20% off. When you get to the till the cashier tells you that she will give you another 15% off of the sale price. What was the price with the regular discount? What is the price with the additional 15% discount?

4. Lou-Anne finds a sale on DVD sets. The set she wants is normally \$37.50 but is now on sale for \$15.00. What percent discount is Lou-Anne getting?

Group 2: Using Percents to Calculate Sales Tax

Important things to know about Sales Tax:

Example Questions

5. Clare wants to buy a leather jacket. The regular price is $288.60 and the GST is 6%. How much will jacket cost Clare?

6. Bobby wants to buy a new stereo that for $48.50. He lives in BC and has to pay 6% GST and 7% PST. How much will the stereo cost?

7. Hockey Sticks are regularly priced at $29.99 in Ontario where the PST is 8% and the GST is 6%. How much will the hockey stick cost in Ontario including sales tax?

8. Lou-Anne wants to buy a DVD for $18.99. When she gets to the till she ends up paying $21.84. What percentage did she pay in sales tax?

Group 3: Using Percents to Calculate Tips

Important things to know about Tips:

Example Questions

9. Clare takes her family out to dinner and the bill comes to $53.85. She wants to leave a 15% tip. How much should she tip? How much money should she leave?

10. Bobby goes to a restaurant where his food is burnt, and the waiter accidentally dumps a pitcher of soda on him. He decides to only tip 8%. If his bill came to $15.90, how much tip should he leave?

11. Josie is out for lunch with sister and offers to pay the bill. Just as she reaches for the bill a glass of water spills and smudges the numbers on the bill. All Josie can see now is that the GST for the meal came to $2.25. If she wants to leave a 15% tip, how much should she leave, and how can she use the GST to help her calculate the tip?

12. Lou-Anne goes out for wings with 7 of her friends. The bill comes to $114.76. Each person in the group pitches in $2 for the tip. Did the group leave a good tip or a bad tip? Explain?

Group 4: Using Percents to Calculate Percent Increase or Decrease

Important things to know about Percent Increase or Decrease:

Example Questions

13. Clare finds a shampoo bottle that advertises that it has 30% more than the old bottle. If the old bottle had 650grams of shampoo, then how much more does the new bottle have?

14. Bobby finds his favourite brand of ice cream has increased in price from $3.95 to $4.35. What is the price increase?

15. There were 20 people in Anna's aerobics class, three people dropped out. What is the percentage decrease in members?

16. Lou-Anne bought a container of Halloween candy for $4.99. The regular price was $5.49. What is the percentage decrease in the price?

Activity 9 – Sales Tax and Discounts

1. Complete the following table.

Item	Price	Percent Discount	Discount	Sale Price	Percent Paid $\frac{\textit{Sale Price}}{\textit{Original Price}}$
Football		10%			
Clown Doll		15%			
Ring Stack		25%			
Toy Car		30%			
Chess Game		50%			
Top		5%			

2. Add the discount and the sale price together for each toy. What do you notice? Why is this?

3. Add the percent discount and the percent paid together for each toy. What do you notice? Why is this?

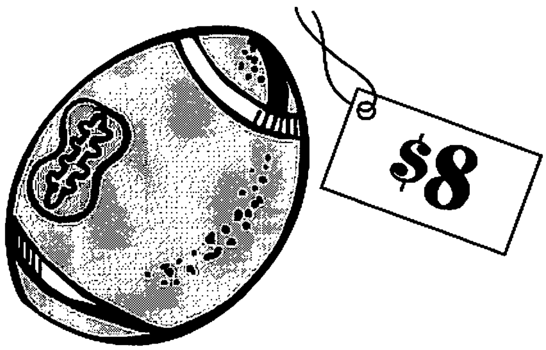
4. Complete the following table.

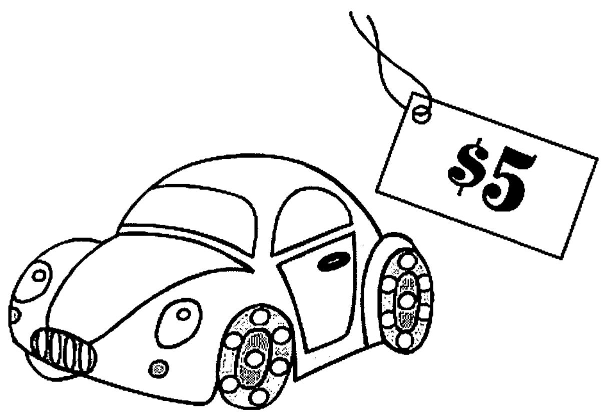
Item	Price	Sales Tax	Sales Tax	Total	Percent Paid $\frac{\text{Total Price}}{\text{Original Price}}$
Football		5%			
Clown Doll		6%			
Ring Stack		7%			
Toy Car		4%			
Chess Game		5%			
Top		5%			

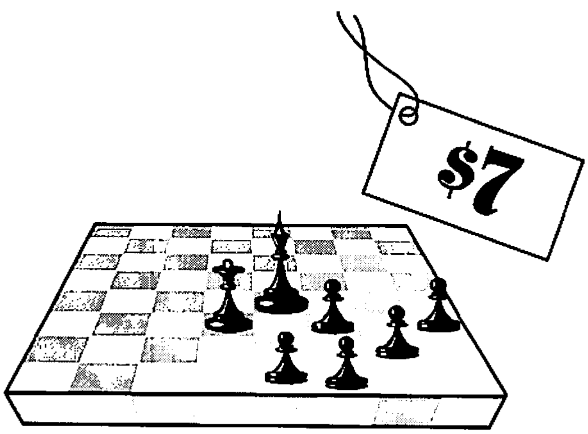
5. Multiply the original prices for the football, chess game, and top by 1.05. What do you notice? Why is this?

6. How could you use what you noticed to calculate the total for the items? What would you multiply by to calculate the total for the clown doll, the ring stack, and the toy car?

Toy Purchases







Word Problems **Name:________________**

1. The Ascot Hotel has 180 rooms. From Monday to Thursday, the hotel is 70% full. From Friday to Sunday, the hotel is 45% full. How many rooms does the hotel rent each week?

2. In its first year, an infant averages 40% of each day awake, 25% of each day sleeping and dreaming, and 35% of each day sleeping without dreaming. How many hours does the infant spend on each activity during its first year?

3. In 1991, 21% of the people in Canada were under 15 years of age. Another 48% were aged 15 to 44, and 31% were 45 or older. Predict the number of people in each age group in a town of 5000 people.

4. A cheeseburger is about 11% fat by mass. How much fat is in a 118-g cheeseburger to the nearest gram?

5. In the 2001-2002 NHL hockey season, Jerome Iginla took 311 shots. He scored on 16.7% of them. How many goals did he score?

6. The Mackenzie River, 4241 km long, is Canada's longest river. The St. Lawrence River is 72% of the length of the Mackenzie River. How long is the St. Lawrence River to the nearest kilometer?

7. Stanley answered 75% of the 60 questions on a science test correctly. He answered 60% of 75 questions on a math test correctly. On which test did he get more correct answers? Explain.

8. Including the end zones, the Canadian football field is about 146 m long and about 60 m wide. The American football field is 25% shorter and 18% narrower than the Canadian Field. What are the dimensions of the American football field?

9. According to a survey, 15% of Canadian adults run regularly. In 2001 there were 30 007 094 Canadians aged 18 and over. How many of these people ran regularly?

10. To be a bestseller, at least 5000 copies of a hardcover book or 30 000 copies of a paperback must be sold. According to Cynthia Good, publisher of Penguin Books Canada Ltd.., "... the usual royalty for authors is 10% of the selling price for a hardcover and 8% for a paperback. So, to make it easy, if it's a $30 hardcover they would make $3 a book." How much would a an author receive for:
    a) a $32.95 that sells 5500 copies?
    b) a $10.95 paperback that sells 45 000 copies?

Word Problems

1. If a pair of pants in Alberta were $83.88 and the sale price was 35% off, how much would you pay for the pants? Then calculate the G.S.T., discount and the total price.

2. I went to the movies in Labrador, the price to go in was $5.00. I bought a small popcorn which was $5.25 and a small pop which was $3.50. There was a sale price of 25% off of the door price which was $5.00. What is the total you would have to pay including G.S.T. and P.S.T. which is charged for everything?

3. You buy an item at Reitmans and the price is $74.87. It is on sale for 75%, how much will you pay? What is the discount? The sale price?

4. You want to buy a huge cotton ball for $69.00. As you walked up to the store you found a coupon to save 22%. You are in Alberta but they are short on cotton so you need to chip in $2.00 extra for support after G.S.T. How much do you pay?

5. If you wanted to buy clothes for $75.00 at Reitmans and they were 15% off but it was Friday and on Friday you get clothes at 20% off. Calculate the sale price, G.S.T. and total cost.

6. If an item is bought in Saskatchewan and it cost $197.89 and is 25% off. How much does it cost altogether?

7. Drew and Shawn went to "Super Sports" on their holiday in BC. Shawn saw a hockey stick he wanted and it cost $21.00. Drew offered to pay the G.S.T. (because he's a good guy!). When they got to the register, the clerk said that they were the 1000th customer so they got 50% off their purchases. How much did Shawn pay and how much did Drew pay? What was the total price?

8. Superstore in Medicine Hat regularly have 1 litre of Sunripe Juice at $1.11. The sale price is $.99. What percent does the consumer save? If you bought five juices how much would you save before G.S.T.? After?

9. You buy a sweater in Saskatchewan for $46.39 at 30% off. How much is the discount, sale price and total cost?

10. Hockey sticks are usually priced at $84.85. They are on sale for 40% off. Ryan, who lives in Alberta wants his mom to buy him the hockey pads. What is the sale price?

11. You buy a $111.43 jacket in Newfoundland. There is a 43% discount as well as a 10% discount for being first customer at the store. How much will you pay altogether?

12. You buy an item for $47.99. The P.S.T. is 13%. The item you buy is on sale for 40% off. What is the sale price? Discount? P.S.T.? G.S.T.? Total?

13. Ringette sticks are regularly priced at $29.89. They are on sale for 25% off. Lindsey wants her mom to buy 2 sticks. She is buying her sticks in Ontario. What is the total cost, discount, G.S.T. and P.S.T.?

14. You bought an item in Quebec. The item is cool so it cost a lot. The price was $61.69 and the item was on sale for 80% off. How much will you pay?

15. You have five hundred dollars. You drive to Alberta but get stopped by the cops for drinking and driving. You have to pay $236.22. You finally make it to the Brick in Calgary. Everything is 11% off. There is a robbery in the store and the robber takes $62.23 from you. After that you buy a chair for $73.27 plus G.S.T. How much change do you get back from what you have left?

16. An item was bought in a store for $97.63. Above the price it said it was 45% off. When the man looked at the receipt he found out he saved 51%. What was the total cost he paid for the item including G.S.T.?

17. An item is bought in Quebec. It costs $49.99. This item is on sale for 25% off. How much is the sale price? How much is the total cost?

18. Susie went to a sporting good store and she saw a snow board for $300.00 with a sign beside it saying 35% off. What is the discount on this item?

19. You want to buy a radio. It is $70.99 with a sale of 50% off. You want to find out the discount that you are getting, sale price and total.

20. You are buying 2 cd's at $30.00 each, a cd stack at $15.00 and a 5 disc cd player at $520.55. This store is moving and so they are having a liquidation sale and everything is on sale at 30% off. You are buying all this in Quebec. What is the total cost of everything?

21. If an item costs $24.99 and you pay 75% what is the total cost of the discount and the G.S.T.?

22. You have $35.00 and 4 items are priced at $12.50, $10.50, $5.00 and $2.50. With 5% off each item what is the total cost?

23. A store in Alberta has a number of items at 40% off. Nancy bought a bike at $149.99, a camera for $63.79 and a radio for $16.95. Calculate the discount, the sale price and the total cost altogether.

24. Cora bought a watch that was $12.46 but on sale for 37% off. She also got a $62.12 painting, that was on sale for 21% off. She got these in Ontario. What is the total cost?

Math 8 **Name: ________________**
Christmas Gift List

You have $500 to spend on 10 Christmas Gifts. You must identify 10 people you are choosing gifts for and then search through the flyers to identify what you would like to give them. At least five of your gifts must be on sale. Determine the sale price, the percent discount, and the total including GST for each item. You may not spend more than your budget, but anybody who comes within $1 of the $500 budget will receive a prize. Use the table below to record your gift ideas.

Gift For:	Item & Store	Regular Price	Percent Discount	Sale Price/Discount Calculations	Sale Price	Total Calculations	Total Including GST

Gift For:	Item & Store	Regular Price	Percent Discount	Sale Price/Discount Calculations	Sale Price	Total Calculations	Total Including GST
						Total Budget Spent	
						Total Budget Remaining	

Christmas List Project

- You have been given $500 to spend on Christmas gifts for 10 people.

- You must provide:
  - i) a list of the people you are buying for.
  - ii) a picture from a flyer/catalogue of what you decided to get him/her with the price on it (at least four items must be on sale).
  - iii) a calculation of the **amount** of discount you received and the **rate** of discount (%) that this works out to for the four items which are on sale (no ads where the % discount is given).
  - iv) a calculation of the sales tax you will pay on each item and its final cost to you (no "GST included").
  - v) a summative list with your $500.00 at the top and the cost of each item bought subtracted consecutively from it.

- You must get as close to having no money left over as possible.

- Gifts cannot be duplicated, nor can you give money or buy for yourself.

- Your project's calculations must be **typewritten** and pasted on construction paper/cardboard. The headings for the chart should be: "Name, Gift, Regular Price, Sale Price, $ Discount, % Discount, $GST, Cost, Balance".

- A good project will:
  - i) be neat, well-organized, and create a good impression.
  - ii) have accurate calculations.
  - iii) include reasonable gifts for each person on your list.
  - iv) use up all your money.

- class time will be given for this project.

- This is an individual project. Marks for any group work will be divided among the collaborators.

- Lates will **not** be accepted.

- This project will be worth 45 marks and is due on Nov 30th.

In general

Christmas List Project Marking Criteria

Sales tax/cost calculations on regularly-price items (2 marks each x 6 gifts)	/ 12
Discount amount/ sales price/ sales tax/cost of item (4 marks each x 4 gifts)	/ 16
Presentation (appealing, neat, layout, decoration)	/ 5
Pictures with prices	/ 5
Money left over	/ 5

$5.00 or more	0
$4.00 to $5.00	1
$3.00 to $4.00	2
$2.00 to $3.00	3
$1.00 to $2.00	4
Less than a $1.00	5

Right on the money at $500.00 - Bonus 5 marks

Remembering name/ date	/ 2
Total marks	/ 45