Planning for the New Grade 6 Math

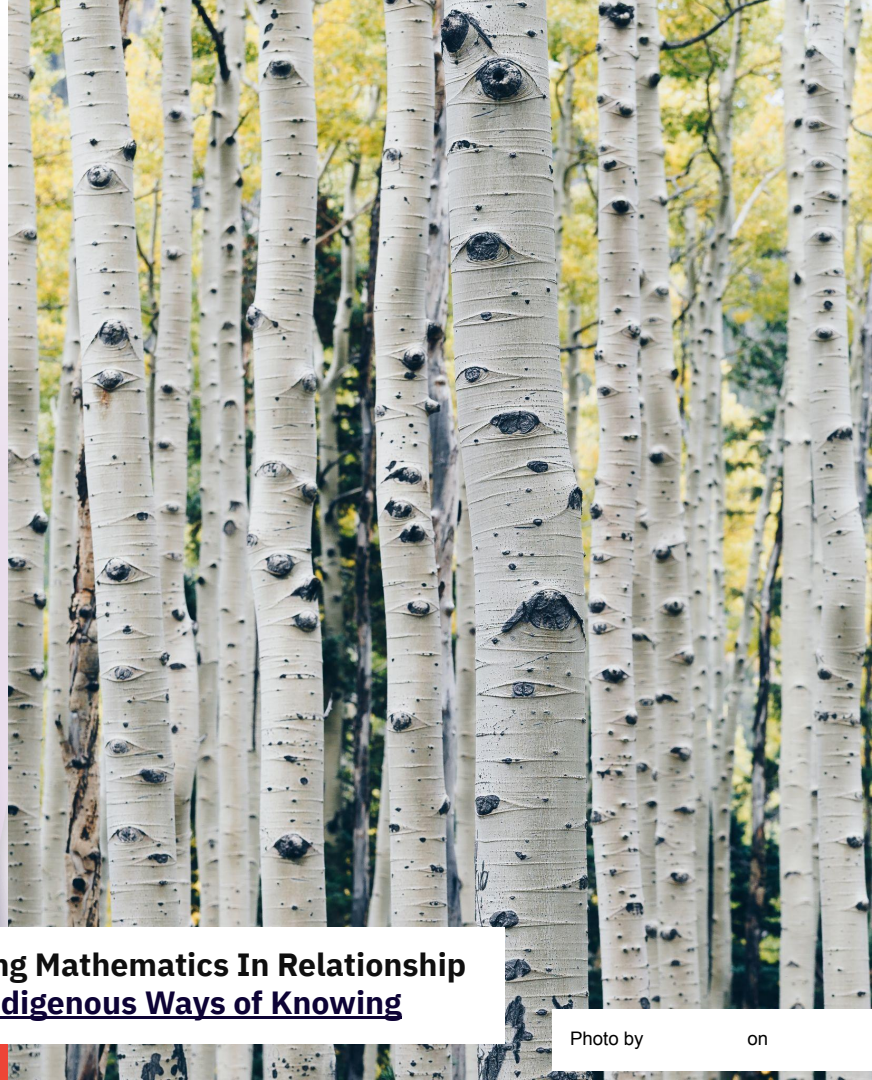
Provincial Session 4
(February, March, April)

February 21, 2024

pcs AlbertaRegionalConsortia

Land Acknowledgement

In the spirit of reconciliation, we want to acknowledge that this gathering is taking place on traditional lands across the province of Alberta, home to many diverse First Nations, Métis and Inuit peoples. We acknowledge that this land is a traditional meeting ground giving voice to its original peoples and the story of creation of this country in a way that history has forgotten.



Teaching Mathematics In Relationship with Indigenous Ways of Knowing

Photo by on

Make It Visible...Make It Real

On our agenda today:

- Fractions - Relate to quotients - 6N5
- Add and subtract fractions with unlike denominators - 6N6
- Multiplying a fraction by a natural number - 6N7
- Students analyze expressions and solve algebraic equations - 6A1.2 (Students should have completed and continue to work with integer operations before solving equations using alge tiles).
- LARGE Numbers!

February	March	April
	Number: Quantity is measured with numbers that enable counting, labelling, comparing and c	
6N5 Students relate fractions to quotients.(begin with unit fractions for money, move to equivalent fractions/decimals for money;stay with common denominator initially) • Fractions represent quotients in equal-sharing situations. • All equivalent fractions represent the same quotient 6N6 Students add and subtract fractions with denominators within 100. • Fractions with common denominators have the same units. • Any numbers with the same unit can be compared, added, or subtracted. * Review of math 12 x 12 facts with different strategies should be ongoing.	6N7 Students interpret the multiplication of natural numbers by fractions.(begin with familiar unit fraction in money;review counting by unit fractions) • Multiplication does not always result in a larger number (model several examples in real life before showing algorithms). • Multiplication of a natural number by a fraction can be interpreted as repeated addition of the fraction. • Multiplication of a fraction by a natural number can be interpreted as taking part of a quantity * Review of math 12 x 12 facts with different strategies should be ongoing	6N8 Students apply equivalence to the interpretation of ratios and rates. • All equivalent ratios express the same proportional relationship (use real world examples upfront). • A rate can be used to extend a given proportional relationship to different quantities. * Review of math facts with different strategies should be ongoing Leave for April
	Algebra: Equations express relationships between quantities.	
6A1.2 Students analyze expressions and solve algebraic equations. • Algebraic properties ensure equivalence of algebraic expressions.		6A1.3 Students analyze expressions and solve algebraic equations. • Algebraic expressions on each side of an equation can be simplified into equivalent expressions to facilitate equation solving. (starting with manipulatives only)

Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6
"Sharing"	In a quantity partitioned into two equal groups, each group represents one-half of the whole quantity. In a shape or object partitioned into two identical pieces, each piece represents one-half of the whole. Symmetry	"Quarters" Model a unit fraction.. (10 or fewer parts) Compare unit fractions of the same/different wholes. Make the whole from a **unit fraction.**	Fractions are numbers between natural numbers. Fractions can represent part-to-whole relationships. A **unit fraction** describes the size of the equal parts of a fraction. The size of the parts and the total number of equal parts in the whole are inversely related. (Money)	There are infinitely many equivalent fractions that represent the same number. Exactly one of infinitely many equivalent fractions is in simplest form. (GCF) Decimal numbers that terminate (do not repeat) are fractions with denominators of 10, 100, etc. Fractions, decimals, and percentages can represent the same part-whole relationship.	Fractions allow counting and measuring **between whole** quantities. Improper fractions and mixed numbers that represent the same number are associated with the same point on the number line.	Model an equal-sharing situation in more than one way. Describe an equal-sharing situation using a fraction. Express a fraction as a division statement and vice versa. Convert a quotient from fraction to decimal form using division.

Grade 4	**Grade 5**	**Grade 6**
There are infinitely many equivalent fractions that represent the same number. Exactly one of infinitely many equivalent fractions is in simplest form. (GCF) Decimal numbers that terminate (do not repeat) are fractions with denominators of 10, 100, etc. Fractions, decimals, and percentages can represent the same part-whole relationship.	Fractions with common denominators are multiples of the same **unit fraction**. Properties for addition and subtraction of natural numbers apply to fractions.	Fractions with common denominators have the same units. Any numbers with the same unit can be compared, added, or subtracted. Multiplication does not always result in a larger number. Multiplication of a natural number by a fraction can be interpreted as **repeated addition** of the fraction. Multiplication of a fraction by a natural number can be interpreted as taking part of a quantity.

6N5 Students relate fractions to quotients.(begin with unit fractions for money, move to equivalent fractions/decimals for money;stay with common denominator initially)

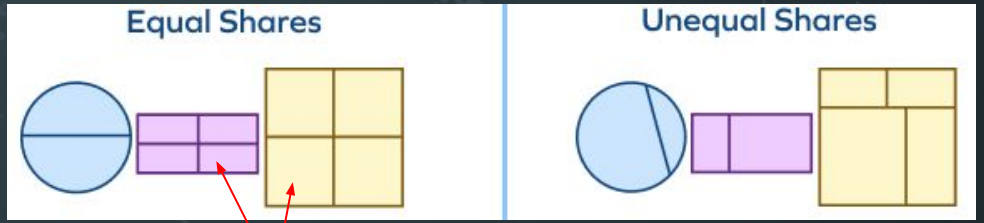
- Fractions represent quotients in equal-sharing situations.
- All equivalent fractions represent the same quotient

How can equal sharing contribute meaning to fractions?

6N5 Students relate fractions to quotients.

Knowledge	Understanding	Skills & Procedures
An equal-sharing situation can be represented by a fraction in which the numerator represents the quantity to be shared and the denominator represents the number of shares. Division can be used to determine an equal share. Division of the numerator by the denominator of a fraction provides the equivalent decimal number.	Fractions represent quotients in equal-sharing situations. All equivalent fractions represent the same quotient.	Model an equal-sharing situation in more than one way. Describe an equal-sharing situation using a fraction. Express a fraction as a division statement and vice versa. Convert a quotient from fraction to decimal form using division.

It is important that students have a context for the 'bar' that exists in a fractions symbolic representation.



$\frac{1}{4}$

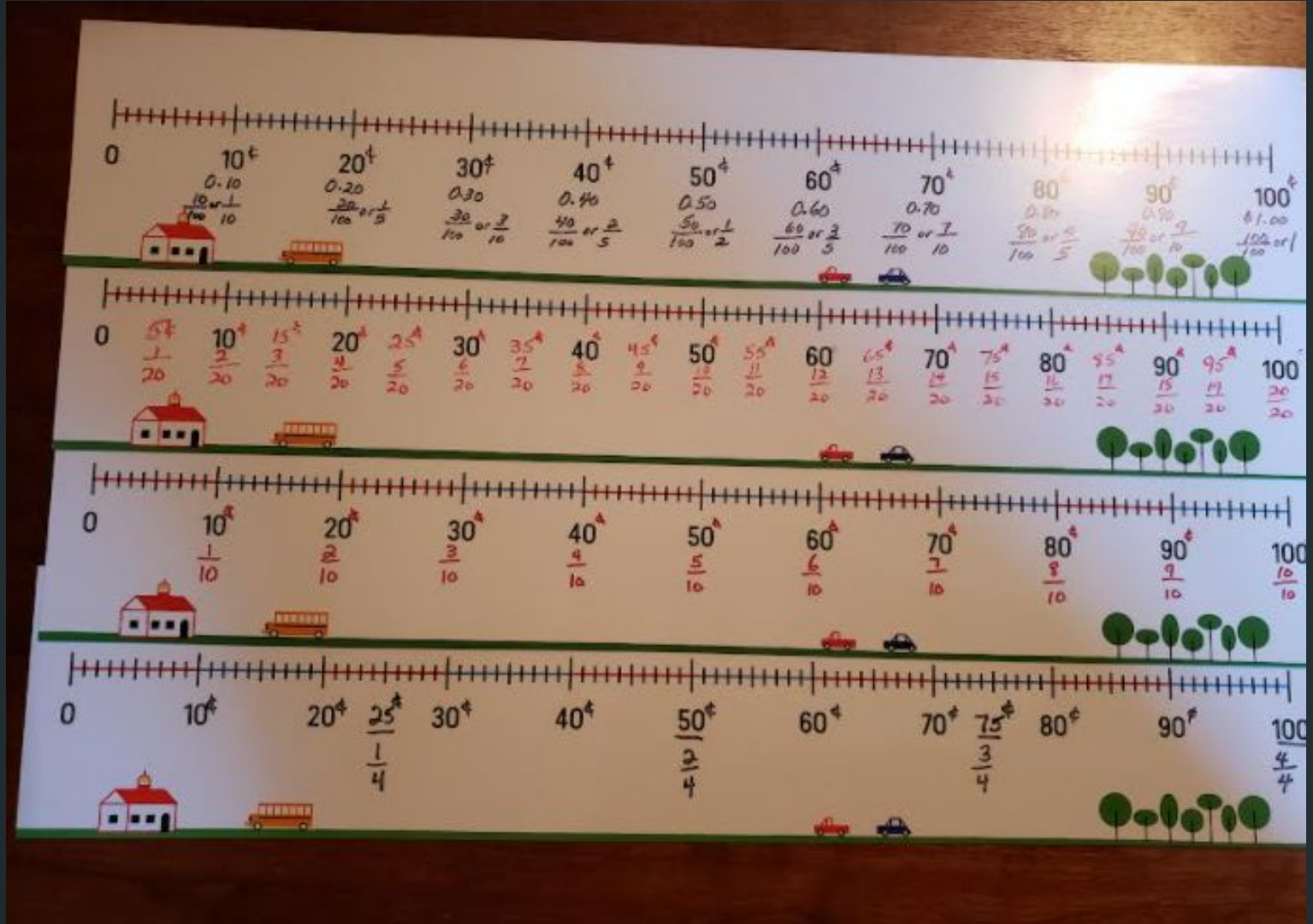
$$\frac{1}{5}$$

Represents a division sign

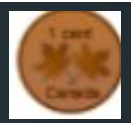
$1 \div 5$

$1.00 was divided in equal shares between 5 people. How much did each person receive?

$1 \div 5 = 0.2$


0
10¢ 0.10 10/100 or 1/10
20¢ 0.20 20/100 or 1/5
30¢ 0.30 30/100 or 3/10
40¢ 0.40 40/100 or 2/5
50¢ 0.50 50/100 or 1/2
60¢ 0.60 60/100 or 3/5
70¢ 0.70 70/100 or 7/10
80¢ 0.80 80/100 or 4/5
90¢ 0.90 90/100 or 9/10
100¢ $1.00 100/100 or 1
0
1/20 2/20 3/20 4/20 5/20 6/20 7/20 8/20 9/20 10/20 11/20 12/20 13/20 14/20 15/20 16/20 17/20 18/20 19/20 20/20
0
1/10 2/10 3/10 4/10 5/10 6/10 7/10 8/10 9/10 10/10
0
10¢ 20¢ 25¢ 1/4 30¢ 40¢ 50¢ 2/4 60¢ 70¢ 75¢ 3/4 80¢ 90¢ 100¢ 4/4

DOLLARS				CENTS	
$1000	$100	$10	$1	dimes	pennies



= 10 ¢

20 ¢ 2 dimes no cents

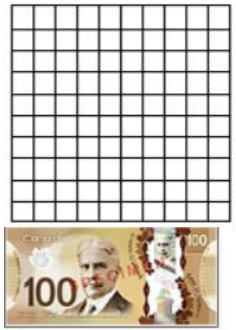
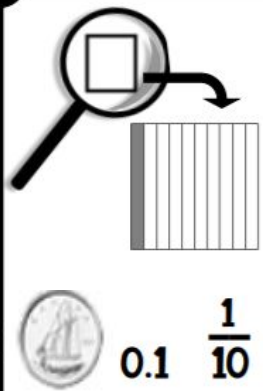
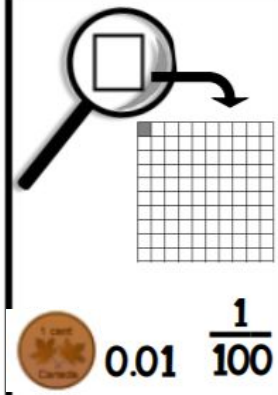
= 1 ¢

2 ¢ no dimes and 2 cents

DOLLARS				CENTS	
$1000	$100	$10	$1	dimes	pennies
____x 1000 =	____x 100 =	____x 10 =	____x 1 =	____x 0.1 =	____x 0.01=
____x 1000 = 1	____x 100 = 1	____x 10 = 1	____x 1= 1	____x 1 = 10	____x 1 = 100
					

Money Place Value Place Mats

Select Place Value Template from the zip file.

Hundreds	Tens	Units	Tenths	Hundredths
 100.00	 10.00	 1.00	 0.1 $\frac{1}{10}$	 0.01 $\frac{1}{100}$

Real World Context


"Fraction is..."
Symbolic
Numerator and Denominator in a Fraction
Numerator
Denominator
Fraction Bar
Fraction
RELATIONSHIPS

6N6 Students add and subtract fractions with denominators within 100.

- Fractions with common denominators have the same units.
- Any numbers with the same unit can be compared, added, or subtracted.

* Review of math 12 x 12 facts with different strategies should be ongoing.

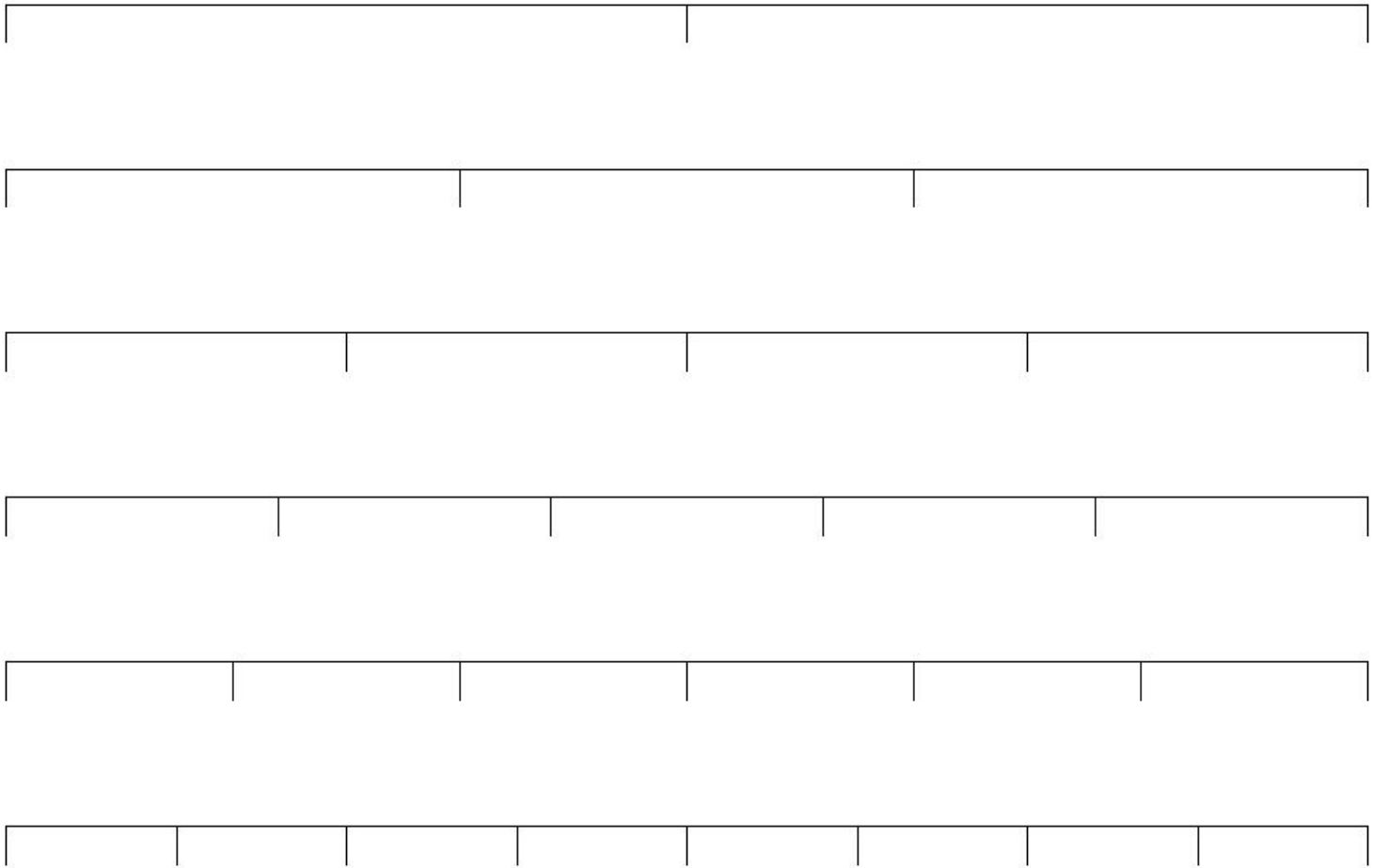
Grade 5 add and subtract like denominators.

How can the addition and subtraction of fractions be generalized?

6N6 Students add and subtract fractions with denominators within 100.

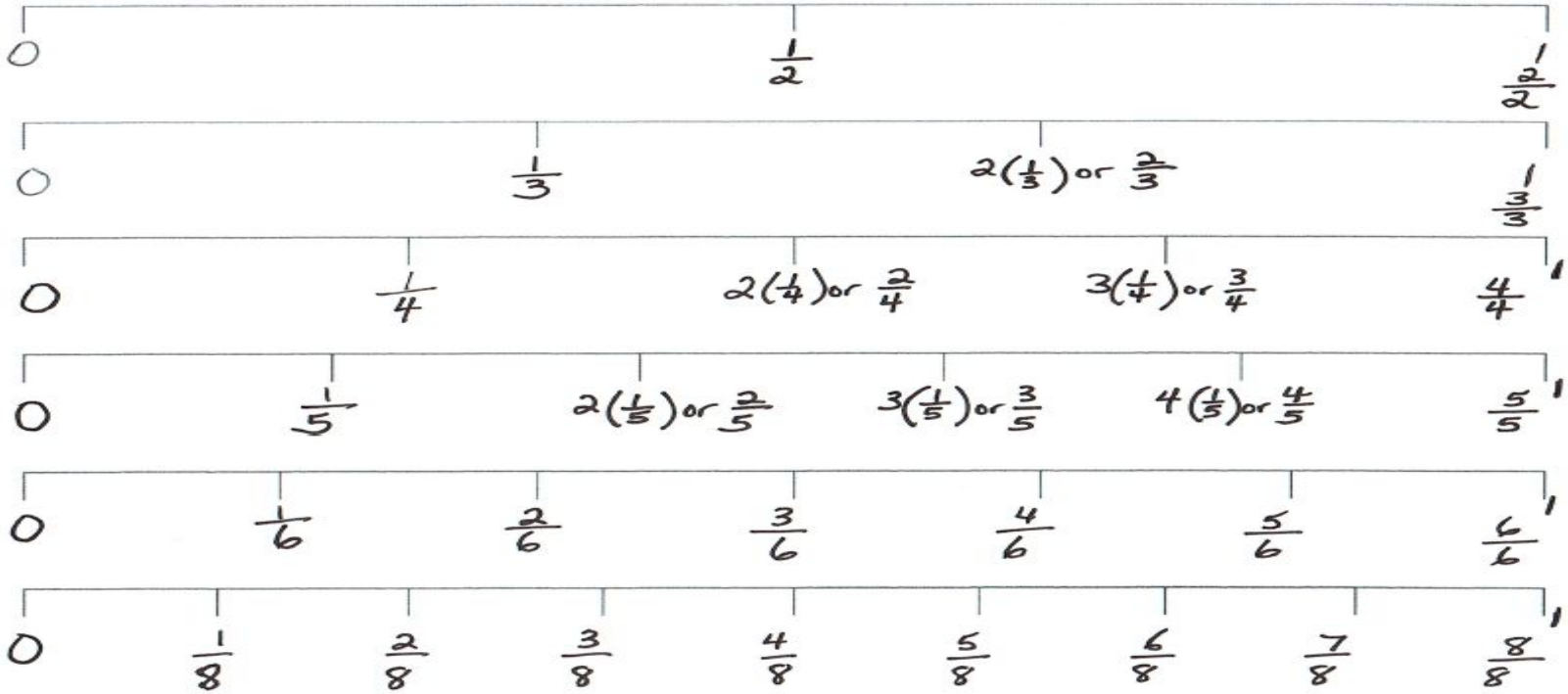
Knowledge	Understanding	Skills & Procedures
Addition and subtraction of fractions is facilitated by representing the fractions with common denominators. Denominators are related if one is a multiple of the other. Multiplication of one denominator by the factor that relates it to another denominator achieves common denominators. The product of the denominators of two fractions provides a common denominator.	Fractions with common denominators have the same units. Any numbers with the same unit can be compared, added, or subtracted.	Recognize two fractions with related denominators. Determine the factor that relates one denominator to another. Express two fractions with common denominators. Add and subtract fractions. Solve problems involving addition and subtraction of fractions.

Provide the "why" behind the "how" first. Revisit equivalent fractions.



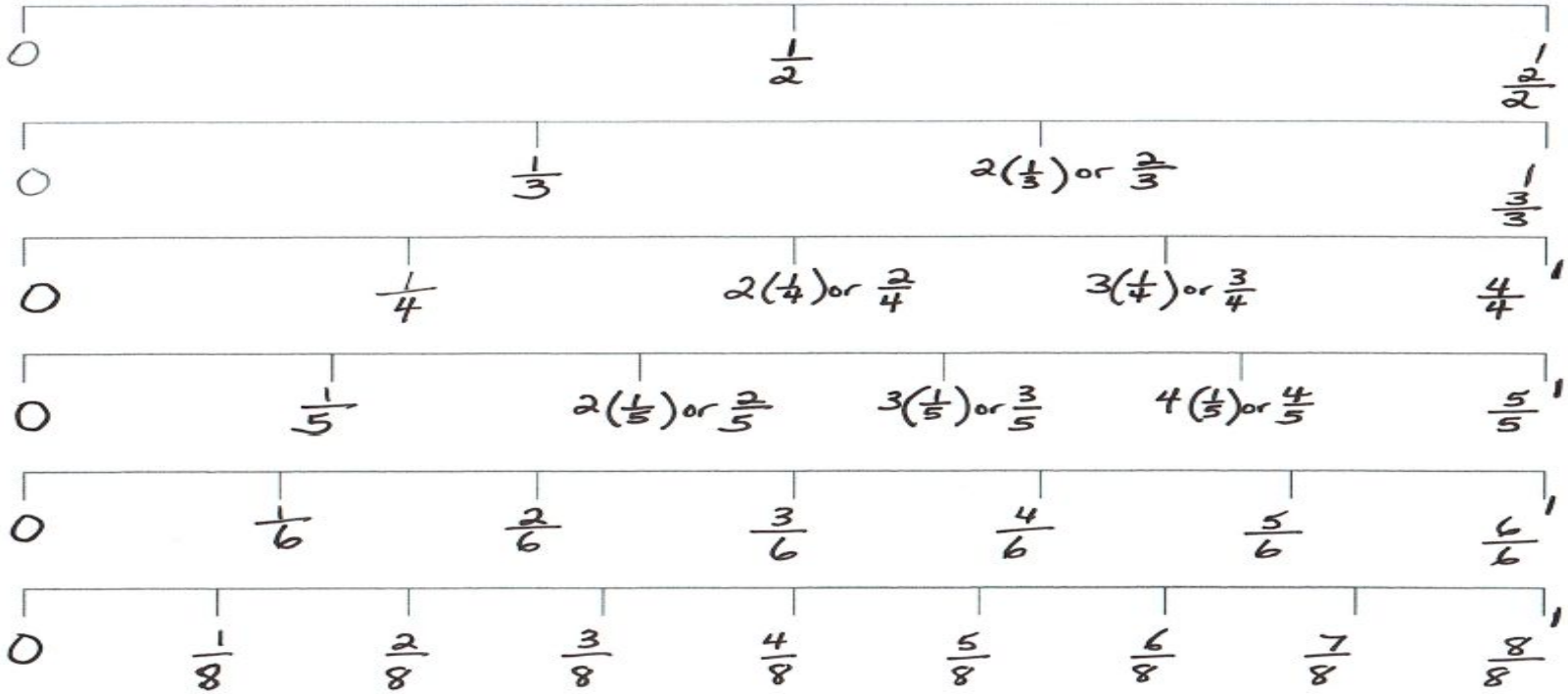
Place 0 down the left side of each number line.
Place 1 down the right side of each number line.

Count by each unit fraction and number each ‘tick’ mark.



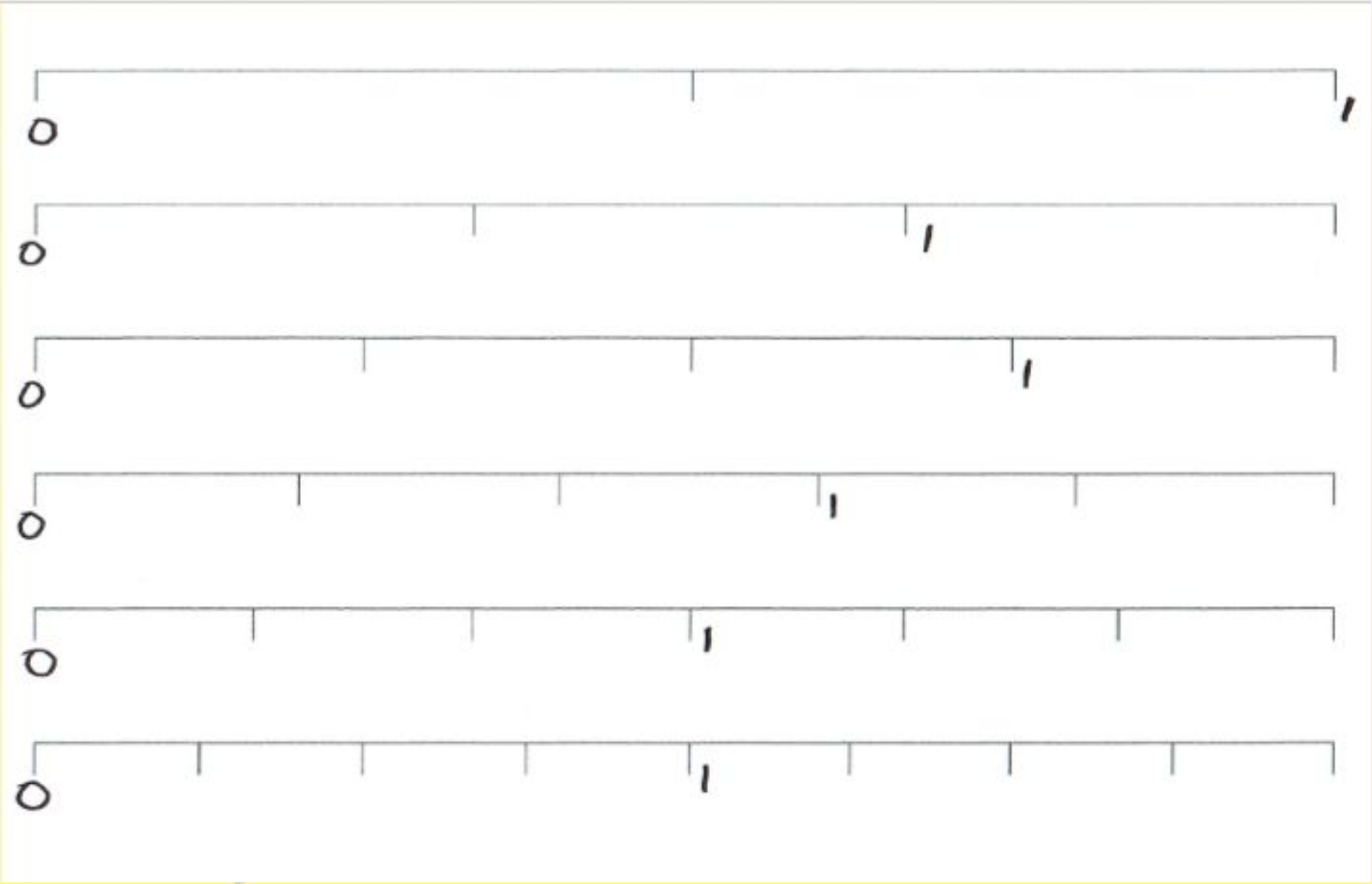
Compare

How can you recognize half?

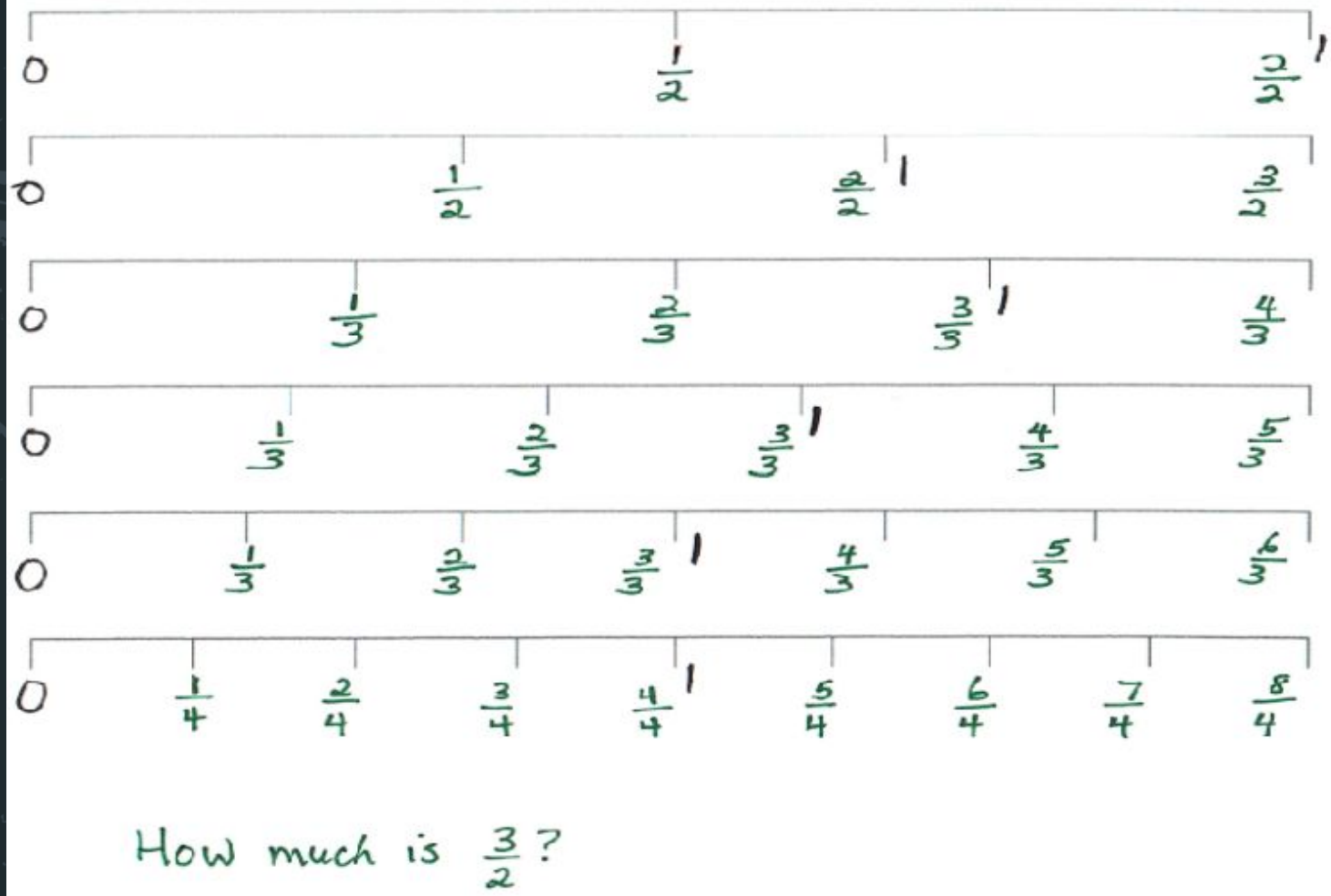


Compare

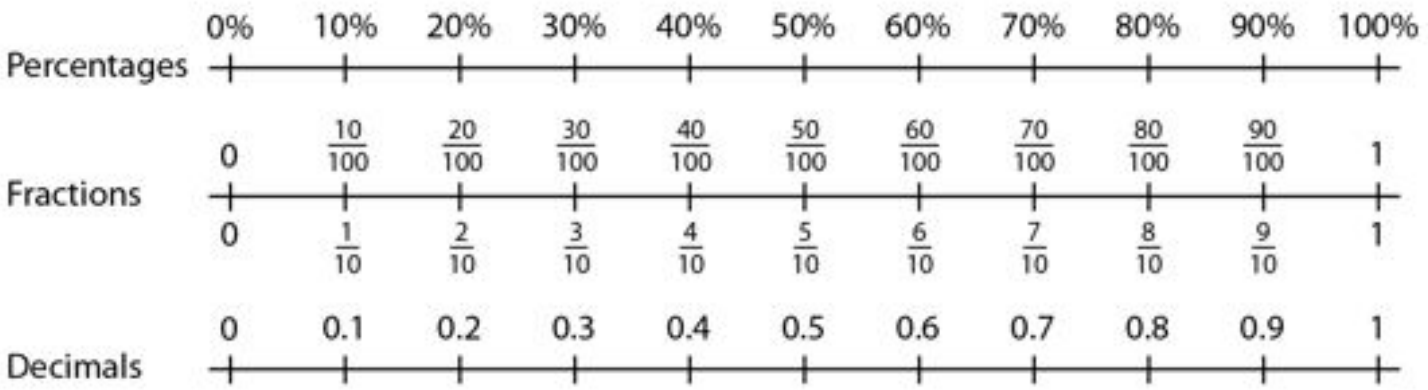
How can you recognize half?



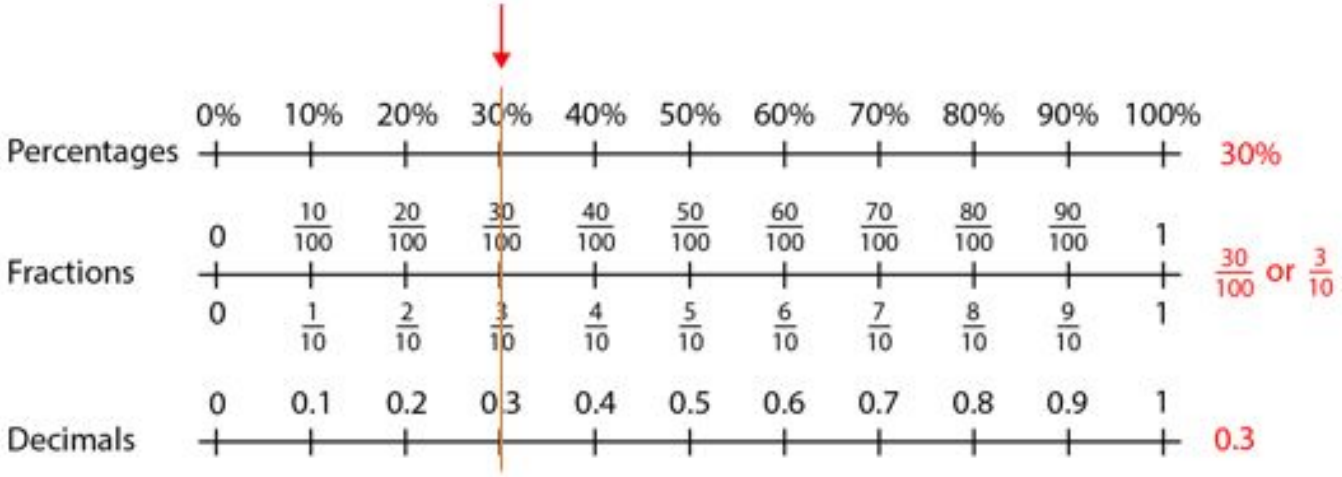
Surface
Deep
Transfer



Would students know where to put the benchmarks on any number line?



If these are dollar amounts, students make a real connection!



Students need to understand

- equivalent fractions and their meaning
- modelling
- factors and multiples

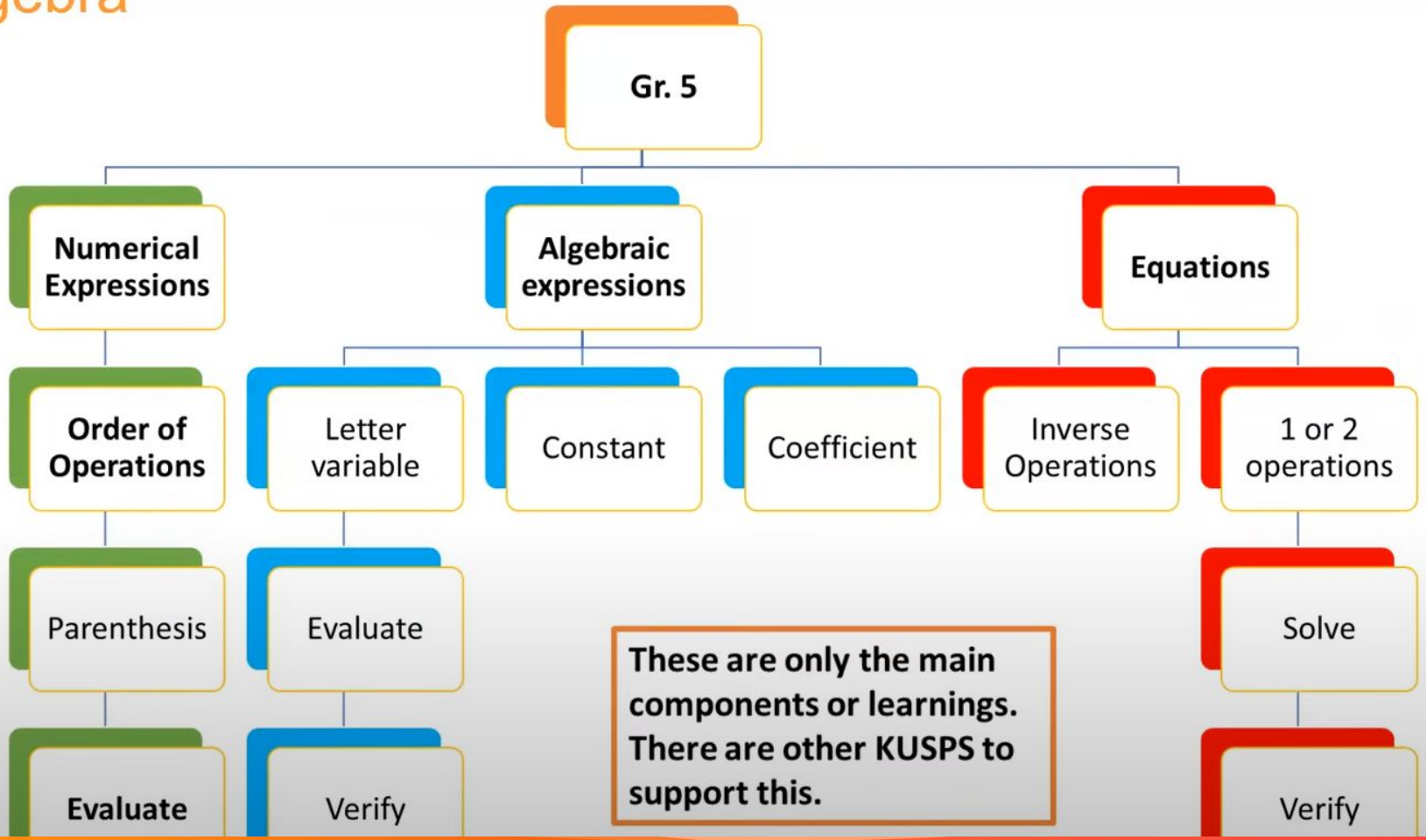
Fraction Set Cards
(Card decks created by Beth Edwards, Junior Division Teacher Consultant, Grand Erie District School Board.. Ontario Ministry of Education)

Fraction Strips (mathies)

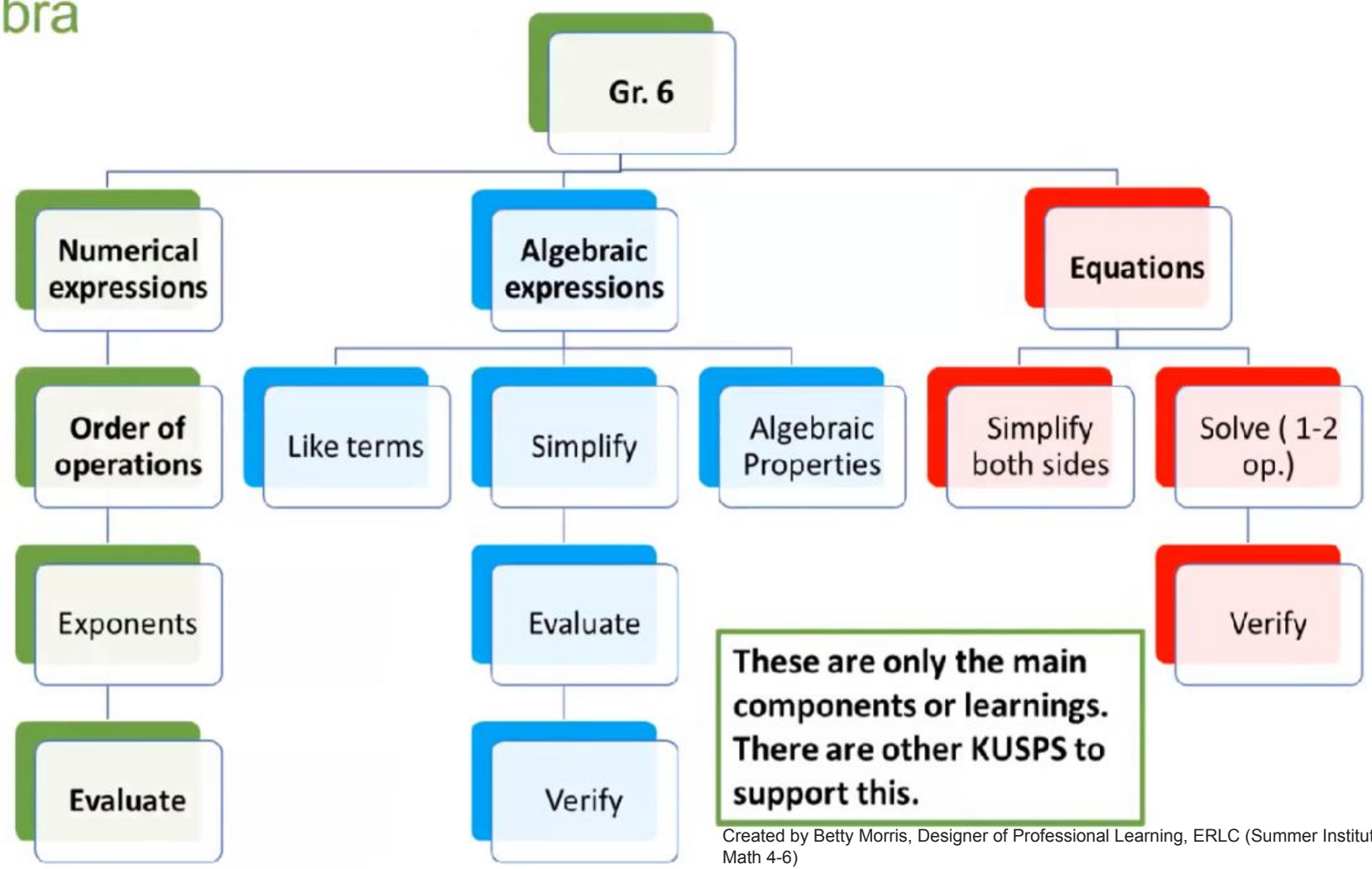
Adding and Subtracting with Unlike Denominators
Includes a brief example of 6N7 - Multiplying a natural number by a fraction

How can an understanding of multiplication be extended to fractions?		
6N7 Students interpret the multiplication of natural numbers by fractions.		
Knowledge	Understanding	Skills & Procedures
Multiplication of a natural number by a fraction is equivalent to multiplication by the fraction's numerator and division by its denominator $a \times \frac{b}{c} = \frac{ab}{c}$ Multiplication by a unit fraction is equivalent to division by its denominator. $a \times \frac{1}{b} = \frac{a}{b}$ The product of a fraction and a natural number is the fraction with a numerator that is the product of the numerator of the given fraction and the natural number a denominator that is the denominator of the given fraction. $\frac{a}{b} \times c = \frac{ac}{b}$	Multiplication does not always result in a larger number. Multiplication of a natural number by a fraction can be interpreted as repeated addition of the fraction. Multiplication of a fraction by a natural number can be interpreted as taking part of a quantity.	Relate multiplication of a natural number by a fraction to repeated addition of the fraction. Multiply a natural number by a fraction. Relate multiplication by a unit fraction to division. Multiply a natural number by a unit fraction. Model a fraction of a natural number. Multiply a fraction by a natural number. Solve problems using multiplication of a fraction and a natural number.

Algebra


Gr. 5
Numerical Expressions
Algebraic expressions
Equations
Order of Operations
Letter variable
Constant
Coefficient
Inverse Operations
1 or 2 operations
Parenthesis
Evaluate
Solve
Evaluate
Verify
Verify
These are only the main components or learnings. There are other KUSPS to support this.

Algebra



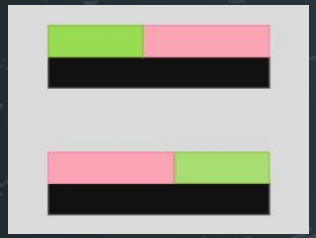
Created by Betty Morris, Designer of Professional Learning, ERLC (Summer Institute Math 4-6)

Grade 3	Grade 4	Grade 5	Grade 6
Knowledge of basic fact 10 x 10	**Order of operations (no parenthesis)** **x, ÷, +, -**	Evaluate numerical expressions with multiple operations (with addition and subtraction in parenthesis)	Evaluate numerical expressions using order of operations and powers
Equality between a number and an **expression** **7 has the same value as 4 + 3** **7 = 4 + 3**	Create **numerical expressions** with multiple operations **3 x 6 +2**	Work with algebraic expressions with a variable, constant, and coefficient **3*n* + 2**	Understand and apply algebraic properties:
Equality between 2 expressions of the same number **3 + 3 = 4 + 2**	Evaluate numerical expressions with multiple operations **5 + 4 – 3**	Evaluate algebraic expressions of form x +6, 2x, x/2, 2x + 6 when given the value of the variable	Simplify algebraic expressions by combining like terms **2 x + 3 x**
Equations can have **unknown values** that can be represented by **symbols** **5 - ◯ = 2**	Understand and apply preservation of equality in an **equation** without an unknown value (hands on) **7 = 7 ⟹ 7 + 2 = 6 + 3**	Using inverse operations to solve an equation, limited to 1 or 2 operations	Solve equations with algebraic expressions on both sides of the equations. limited to 1 or 2 operations
Determine an unknown value of an equation (Using manips, balance, reasoning)	Solve an equation with an unknown value, limited to one operation **7 + ? = 13**	Verify solutions to equations through evaluation	Verify solutions to equations through evaluation

6A1.2 Students analyze expressions and solve algebraic equations.

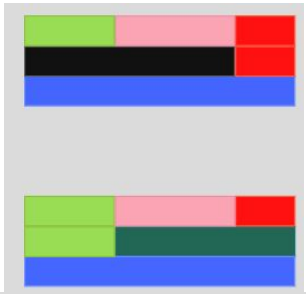
- Algebraic properties ensure equivalence of algebraic expressions.

Cuisenaire Rods can help model this for students



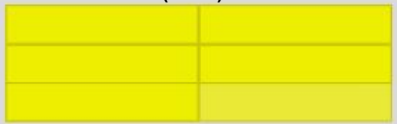
6A1.2 Students analyze expressions and solve algebraic equations.

Knowledge	Understanding	Skills & Procedures
Algebraic terms with exactly the same variable are like terms. Constant terms are like terms. Like terms can be combined through addition or subtraction. The terms of an algebraic expression can be rearranged according to algebraic properties. Algebraic properties include ° commutative property of addition: $a + b = b + a$, for any two numbers a and b ° commutative property of multiplication: $ab = ba$ for any two numbers a and b ° associative property of addition: $(a + b) + c = a + (b + c)$ associative property of multiplication: $a(bc) = b(ac)$ distributive property: $a(b + c) = ab + ac$	Algebraic properties ensure equivalence of algebraic expressions.	Investigate like terms by modeling an algebraic expression. Simplify algebraic expressions by combining like terms. Express the terms of an algebraic expression in a different order in accordance with algebraic properties.



2(3x5)

3(2x5)



March	April
Algebra: **Equations express relationships between quantities.**	
5A1.2 Students interpret numerical and algebraic expressions.	5A1.3 Students interpret numerical and algebraic expressions.
• Algebraic expressions use variables to represent quantities of unknown value. • Algebraic expressions may be composed of one algebraic term or the sum of algebraic and constant terms. (Focus more on evaluating expressions in preparation for equations)	• Equality is preserved by applying inverse operations to algebraic expressions on each side of an equation. • The expressions on each side of an equation will be equal when evaluated using the correct solution.

Let's Consider Variables and Algebraic Expressions... What are they?

Terminology:
Variable
Coefficient
Constant
Expression
Equation

Review of Algebra from Grade 5

Students interpret numerical and algebraic expressions. 5A1.1		
Knowledge	Understanding	Skills & Procedures
Numerical expressions with multiple operations may include parentheses to group numbers and operations. The conventional order of operations includes performing operations in parentheses before other operations.	Numerical expressions represent a quantity of known value. Parentheses change the order of operations in a numerical expression.	Evaluate numerical expressions involving addition or subtraction in parentheses according to the order of operations.
Expressions that include variables are called algebraic expressions. A variable can be interpreted as a specific unknown value and is represented symbolically with a letter. Products with variables are expressed without the multiplication sign. Quotients with variables are expressed using fraction notation. An algebraic term is the product of a number, called a coefficient, and a variable. A constant term is a number. A variable can be replaced by a given number in order to evaluate an expression.	Algebraic expressions use variables to represent quantities of unknown value. Algebraic expressions may be composed of one algebraic term or the sum of algebraic and constant terms. 5A1.2	Relate repeated addition of a variable to the product of a number and a variable. Express the product of a number and a variable using a coefficient. Express the quotient of a variable and a number as a fraction. Recognize a product with a variable, a quotient with a variable, or a number as a single term. Write an algebraic expression involving one or two terms to describe an unknown value. Evaluate an algebraic expression by substituting a given number for the variable.

Start with evaluations: if $x = 5$

x

$x + 2 \rightarrow (5) + 2$

$2x + 1 \rightarrow 2(5) + 1$

$3x + 1 \rightarrow 3(5) + 1$

$2x - 4 \rightarrow 2(5) - 4$

$$\frac{x}{4} \rightarrow \frac{5}{4}$$

X

X + 2 = 4

2x + 1 = 3

3x + 1 = 7

2x - 4 = 4

The process of applying inverse operations can be used to solve an equation. The value of the variable obtained by solving an equation is the solution.	Equality is preserved by applying inverse operations to algebraic expressions on each side of an equation. The expressions on each side of an equation will be equal when evaluated using the correct solution.	Write equations involving one or two operations to represent a situation. Investigate order of operations when performing inverse operations on both sides of an equation. Apply inverse operations to solve an equation, limited to equations with one or two operations. Verify the solution to an equation by evaluating expressions on each side of the equation. Solve problems using equations, limited to equations with one or two operations.

Students do not know integers yet so the inverse operations can be only be subtraction and or division.

Alge-tiles - Didax.com

Solving equations with the tiles will be in the April session.

Equivalence and Compensation - a slide deck that would help bridge the understanding towards a balance model of solving equations.

March 2024

March

Measurement: Attributes such as length, area, volume and angle are quantified by measure.

6M1.1 Students analyze areas of parallelograms and triangles. (link to 6N7)

- The area of a parallelogram can be generalized as the product of the perpendicular base and height.
- The area of a triangle can be interpreted relative to the area of a parallelogram.

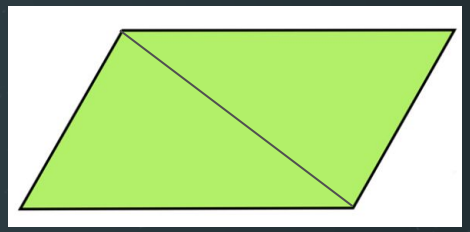
6M1.2 Students analyze areas of parallelograms and triangles.

- Area can be expressed in various units according to context and desired precision.
- Rectangles with the same area can have different perimeters. (models and manipulatives first)

Turning a rectangle into a parallelogram to show area has not changed.

Using A= Base x height

A = ½ (base x height)



Begin with the Grade 5 activity in the previous slides. Cuisenaire rods on a grid would be excellent here.

Grid Paper

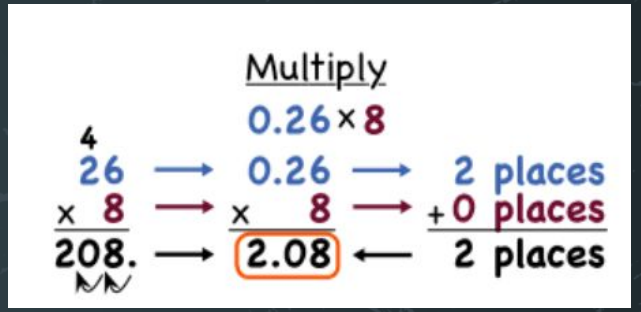
Review Request

How can the processes of multiplication and division be applied to decimal numbers?

6N4 Students apply standard algorithms to multiplication and division of decimal and natural numbers.

Knowledge	Understanding	Skills & Procedures
Standard algorithms are reliable procedures for multiplication and division of numbers, including decimal numbers. A quotient with a remainder can be expressed as a decimal number.	Multiplication and division of decimal numbers is facilitated by standard algorithms.	Explain the standard algorithms for multiplication and division of decimal numbers. Multiply and divide up to 3-digit natural or decimal numbers by 2-digit natural numbers, using standard algorithms. Assess the reasonableness of a product or quotient using estimation. Solve problems using multiplication and division, including problems involving money.

Multiply ignoring the decimals and then place the decimal values into the answer. Why?



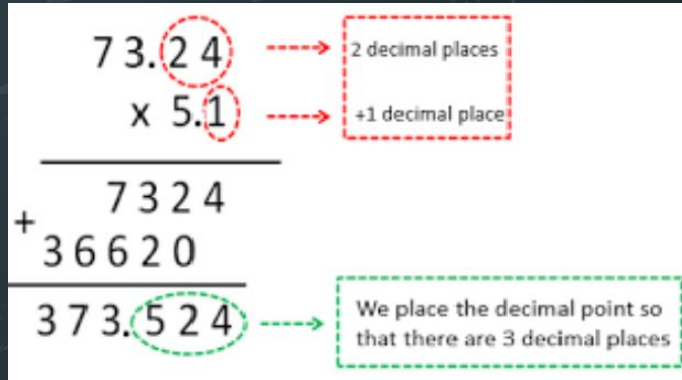
26 ¢ x 8 is close to 8 quarters or 8x25 ¢
$2.00

Resources:
MathLinks 7 Chapter 2: Page 52- 67
Problems Pages 68-73
Review of all operations
Game - Decimal Delights
Financial Literacy Connection - page 79 (Real Life Delights)

Arithmetic Procedures - activities to practice with decimal numbers.

Addition and Subtraction of whole and decimal numbers

CK Middle School Math
-Grade 6 - Chapter 4



Dividing

Place the decimal point in the quotient directly above the decimal point in the dividend.

$$6\overline{)15.6}$$

Divide the same way you would divide with whole numbers.

$$\begin{array}{r} 2.6 \\ 6\overline{)15.6} \\ -\,12 \\ \hline 36 \\ -\,36 \\ \hline 0 \end{array}$$

So, 15.6 ÷ 6 = 2.6.

Math Focus 7 (Nelson) - Unit 3 page 100 - 108 for Multiplying and Dividing Decimals
Problem Solving with decimals - pages 114-116
World of Work - page 117
Review page 121 #9-12 and Page 123 #2, 4.

Math Makes Sense (Pearson) - Unit 3 Pages 100-107 for Multiplying and Dividing Decimals
Problems - pages 121-122 # 9-12 and Page 123#2, 4 for Multiplying and Dividing

Many students would prefer to get "rid of" the decimal.
If we use the powers of 10 we looked at for students who struggle with division, we can make the division easier to understand.

Example:

0.3 x10 = 3
0.34 x 100 = 34

How can we use this to help solve 30 ÷ 0.5?

What would 0.5 x 10 =? (5)
If we multiply the 0.5 by 10 we must do the same to 30
30 x 10 = 300 (bot numbers are 10 times larger)

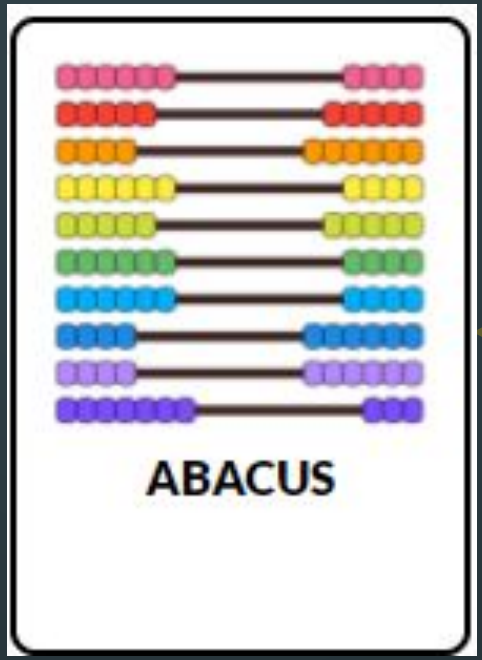
$$\begin{array}{r|l|l} 5 & 300 & 60 \\ & \underline{300} & \\ & 0 & \end{array}$$

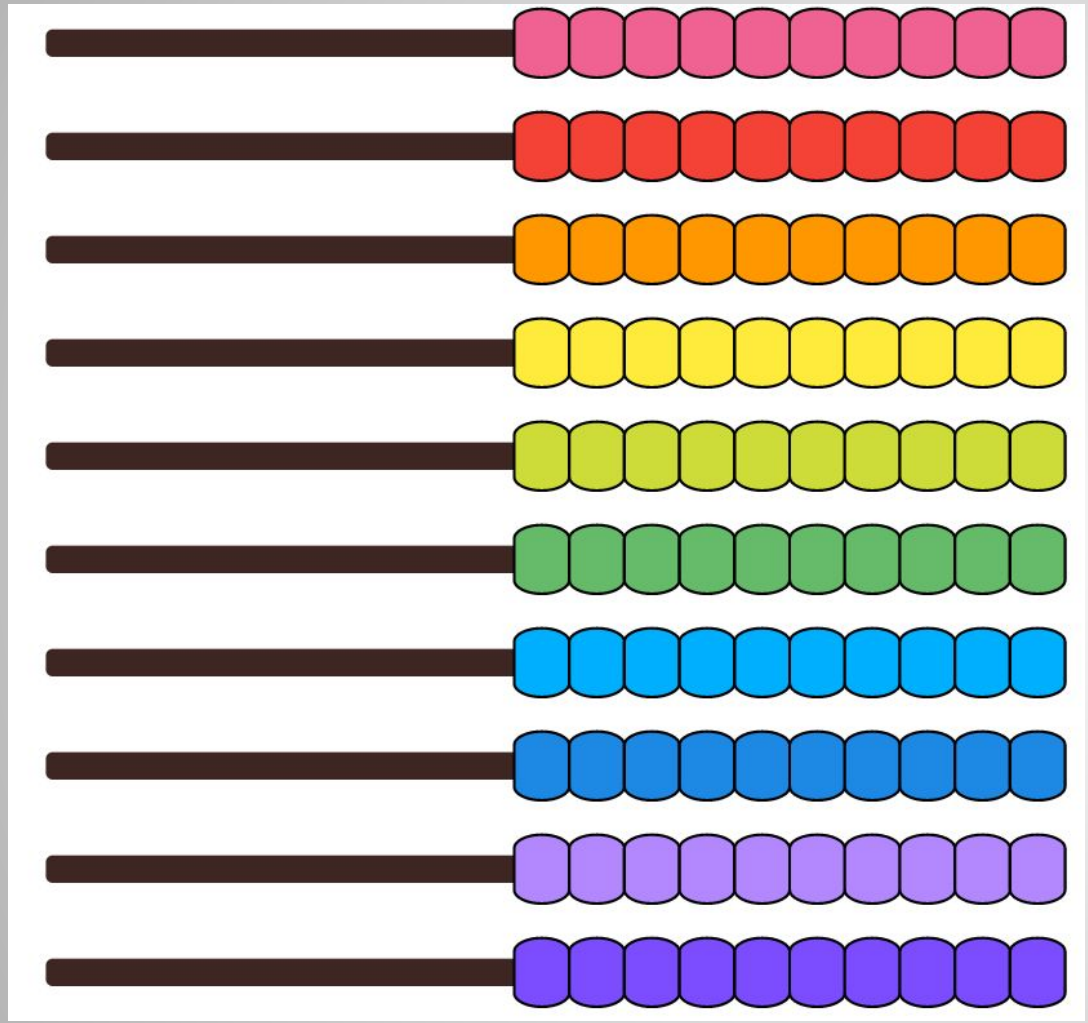
Revisit Large Numbers

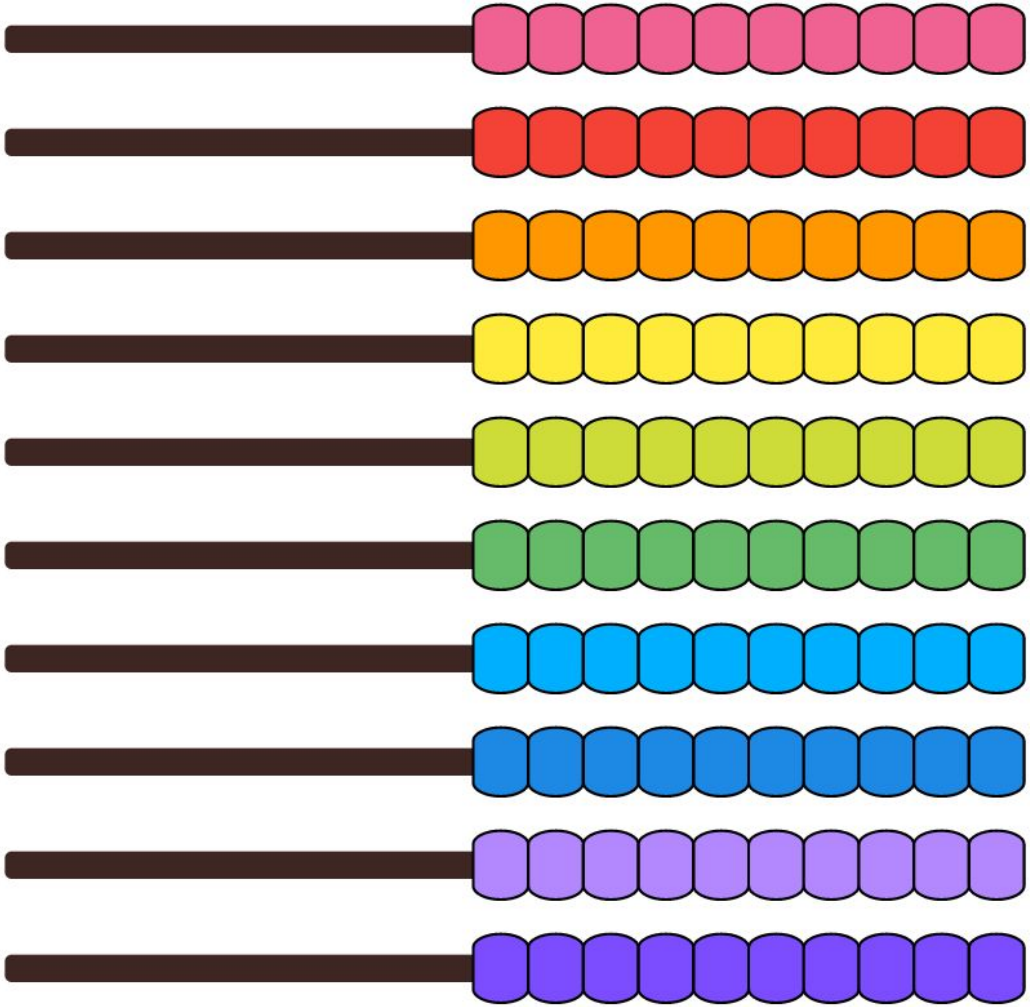
Abacus

Let's add:

12 425 + 6 276



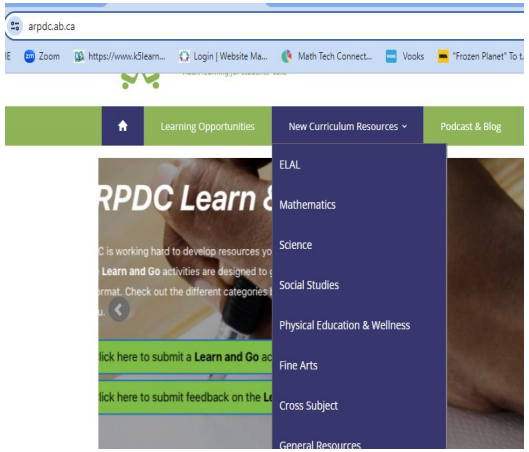

Billion
Hundred Million
Ten Million
Million
Hundred thousands
Ten Thousands
Thousands
Hundreds
Tens
Ones


Hundred Thousands
Ten Thousands
Thousands
Hundreds
Tens
Ones
Decimal
Tenths
Hundredths
Thousandths

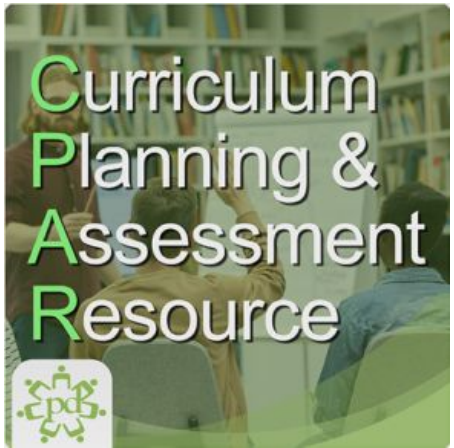
Resource Sites:

www.arpdc.ab.ca

Select Mathematics and Your Grade



CPAR Documents (in Mathematics, Science, ELAL and Computing Science)



Mathematical Verbs

K-3 Mathematics Verbs

Grades 4-6 Mathematics Verbs

Videos unpacking K-3 Mathematics

Videos Unpacking Grades 4-6 Mathematics (3 out of 5 sessions posted)

Thanks!

Any questions?

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- chris.zarski@arpdc.ab.ca
- Connect with your local consortia consultant