Unpacking the New Grade 3 Math

Provincial Group

November 30, 2022

Chris Zarski

Land Acknowledgment

We respectfully acknowledges that we are situated on Traditional lands across the province of Alberta home to many First Nations, including the Cree, Blackfoot, Métis, Nakota Sioux, Saulteaux, Inuit, and many others whose histories, languages, and cultures continue to influence our vibrant community.

Agenda for Today!

- identifying key skills and concepts in the part of our curriculum
- extending **strategies** for 3N2 addition and subtraction using money to support the understanding of place Value
- Using money arrays to help support the learning of math facts - Mastery to 5 x 5 by February,
- Use money to help support the initial work with unit fractions

December	January	February
Number: Quantity is measured with numbers that enable counting, labelling, comparing and operating.		
3N2. Students apply addition and subtraction within 1000.(100-1000) • Addition and subtraction strategies can be chosen based on the nature of the numbers. • Standard algorithms are universal tools for addition and subtraction and may be used for any natural numbers independently of their nature. 3N3.1 Students analyze and apply strategies for multiplication and division within 100. (25) • Quantities can be composed and decomposed through multiplication and division. (5x5) 3N3.2 Students analyze and apply strategies for multiplication and division within 100. • Sharing and grouping situations can be interpreted as multiplication or division.(5x5) • Multiplication and division strategies can be supported by addition and subtraction.	3N2. Students apply addition and subtraction within 1000.(100-1000) • Addition and subtraction strategies can be chosen based on the nature of the numbers. • Standard algorithms are universal tools for addition and subtraction and may be used for any natural numbers independently of their nature. 3N3.1 Students analyze and apply strategies for multiplication and division within 100. (25) • Quantities can be composed and decomposed through multiplication and division. (5x5) 3N3.2 Students analyze and apply strategies for multiplication and division within 100. • Sharing and grouping situations can be interpreted as multiplication or division.(5x5) • Multiplication and division strategies can be supported by addition and subtraction.	3N2. Students apply addition and subtraction within 1000.(100-1 000) • Addition and subtraction strategies can be chosen based on the nature of the numbers. • Standard algorithms are universal tools for addition and subtraction and may be used for any natural numbers independently of their nature. 3N3.1Students analyze and apply strategies for multiplication and division within 100. • Quantities can be composed and decomposed through multiplication and division. 3N3.2 Students analyze and apply strategies for multiplication and division within 100. • Sharing and grouping situations can be interpreted as multiplication or division.(5x5 Mastery) • Multiplication and division strategies can be supported by addition and subtraction. 3N4 Students interpret fractions in relation to one whole. • Fractions are numbers between natural numbers. • Fractions can represent part-to- whole relationships. • A unit fraction describes the size of the equal parts of a fraction. (begin with Unit Fractions linked to money) • The size of the parts and the total number of equal parts in the whole are inversely related.

How can processes be established for addition and subtraction? 3N2 100-1000		
Students apply strategies for addition and subtraction within 1000.		
Knowledge	Understanding	Skills & Procedures
Recall of addition and subtraction number facts facilitates addition and subtraction strategies. Standard algorithms for addition and subtraction are conventional procedures based on place value. Estimation can be used to support addition and subtraction in everyday situations, including • when an exact sum or difference is not needed • to check if an answer is reasonable	Addition and subtraction strategies can be chosen based on the nature of the numbers. Standard algorithms for addition and subtraction may be used for any natural numbers.	Relate strategies for the addition and subtraction of two-digit numbers to strategies for the addition and subtraction of three-digit numbers. Model regrouping by place value for addition and subtraction. Explain the standard algorithms for addition and subtraction of natural numbers. Add and subtract natural numbers using standard algorithms. Estimate sums and differences. Solve problems using addition and subtraction.

Relate
Model
Explain
Add
Estimate
Solve

Number: Quantity is measured with numbers that enable counting, labelling, comparing, and operating.

How can multiplication and division provide new perspectives of number?

Students analyze and apply strategies for multiplication and division within 100. 3N3.1 25 (5x5)

Knowledge	Understanding	Skills & Procedures
Multiplication and division are inverse mathematical operations. Multiplication is repeated addition. Multiplication can be interpreted in various ways according to context, such as • equal groups • an array • an area Division can be interpreted in various ways according to context, such as • equal sharing • equal grouping • repeated subtraction The order in which two quantities are multiplied does not affect the product (commutative property). The order in which two numbers are divided affects the quotient. Multiplication or division by 1 results in the same number (identity property).	Quantities can be composed and decomposed through multiplication and division.	Compose a product using equal groups of objects. Relate multiplication to repeated addition. Relate multiplication to skip counting. Investigate multiplication by 0. Model a quotient by partitioning a quantity into equal groups or groups of a certain size, with or without remainders. Visualize and model products and quotients as arrays. Recognize interpretations of multiplication and division in various contexts.

Compose
Relate
Relate
Investigate
Model
Visualize
Recognize

Numbers can be multiplied or divided in parts (distributive property). Multiplication strategies include • repeated addition • multiplying in parts • compensation Division strategies include • repeated subtraction • partitioning the dividend Products can be expressed symbolically using the multiplication sign, $\times$, factors, and the equal sign. Quotients can be expressed symbolically using the division sign, $\div$, dividend, divisor, and the equal sign. A missing quantity in a product or quotient can be represented in different ways, including • $a \times b = \square$ • $a \times \square = c$ • $\square \times b = c$ • $e \div f = \square$ • $e \div \square = g$ • $\square \div f = g$ A remainder is the quantity left over after division.	Sharing and grouping situations can be interpreted as multiplication or division. Multiplication and division strategies can be supported by addition and subtraction.	Investigate multiplication and division strategies. Multiply and divide within 100. Verify a product or quotient using inverse operations. Determine a missing quantity in a product or quotient in a variety of ways. Express multiplication and division symbolically. Explain the meaning of the remainder in various situations. Solve problems using multiplication and division in sharing or grouping situations.

3N3.2 5x5

investigate
Multiply
Verify
Determine
Express
Explain
Solve

I

Students interpret fractions in relation to one whole. 3N3.4 Unit Fractions

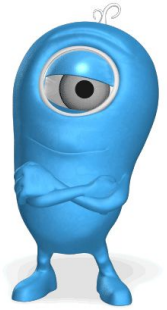
Knowledge	Understanding	Skills & Procedures
The same fraction can represent • equal parts of one whole length, shape, or object • equal groups of one whole quantity • equal parts of each equal group in one whole quantity The name of a fraction describes its composition as a number of unit fractions. Fraction notation, $(\frac{a}{b})$, relates the numerator, a, a number of equal parts, to the denominator, b, the total number of equal parts in the whole. Equal numerators or equal denominators can facilitate the comparison of fractions. A fraction with a numerator that is equal to its denominator is one whole. Each fraction is associated with a point on the number line.	Fractions are numbers between natural numbers. Fractions can represent part-to-whole relationships. A unit fraction describes the size of the equal parts of a fraction. The size of the parts and the total number of equal parts in the whole are inversely related.	Model fractions of a whole quantity, length, shape, or object, in various ways, limited to denominators of 12 or less. Visualize fractions as compositions of a unit fraction. Identify the numerator and denominator of a fraction in various representations. Name a given fraction. Express fractions, including one whole, symbolically, limited to denominators of 12 or less. Relate various representations of the same fraction, limited to denominators of 12 or less. Compare the same fraction of different-sized wholes. Compare different fractions of the same whole that have the same denominator. Compare different fractions of the same whole that have the same numerator and different denominators. Express the relationship between two fractions of the same whole, using $<$, $>$, or $=$. Relate a fraction less than one to its position on the number line, limited to denominators of 12 or less. Compare fractions to benchmarks of 0, $\frac{1}{2}$, and 1.

Model
Visualize
Identify
Name
Express
Relate
Compare
Compare
Compare
Express
Relate
Compare

Compare, Model, Relate, Visualize

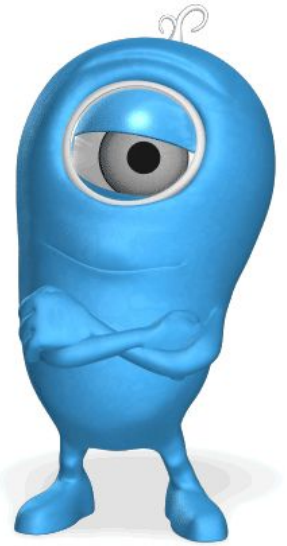
Skill	Concept/Knowledge
Compare	the same fraction of different-sized wholes.
Compare	different fractions of the same whole that have the same denominator.
Compare	different fractions of the same whole that have the same numerator and different denominators.
Compare	fractions to benchmarks of 0, 1/2 , and 1.
Compose	a product using equal groups of objects.
Visualize	and model products and quotients as arrays
Visualize	fractions as compositions of a unit fraction.
	fractions of a whole quantity, length, shape, or object, in various ways, limited to denominators of 12 or less.

Compare, Compose, Visualize, Model



What are the Skills?

Money in application to math facts, place value understanding and position on a numberline.



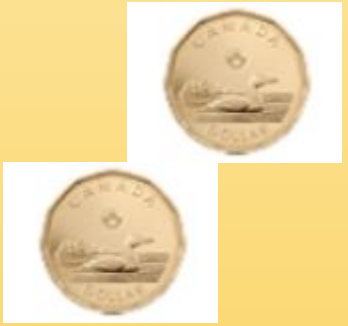
How can we meet these outcomes and leverage money in the process?

TENS

Ones

Answer:

Can you convert and Trade?



How else can we make $68.00

Work on Mastery to 100

Tens

Ones

What number is this?
What number is in the tens place?
How many tens in this number?
How many ones in this number?

What number is this?
What number is in the tens place?
How many tens in this number?
How may ones in this number?

How many tens in...

How many ones?

Work on Mastery to 100

Hundreds

Tens

Ones

What number is this?

What number is in the tens place?

How many tens in this number?

How many ones in this number?

$100	$10	$1
		

$100	$10	$1
		
4	3	4

Extending the Understanding to Place value

Place Value Chart Beginning with Money (Penny, Dime, Loonie, $10, $100, $1000)

 (not legal tender)					

As of January 1, 2021, the $1, $2, $25, $500 and $1,000 bills from every Bank of Canada series are no longer legal tender.

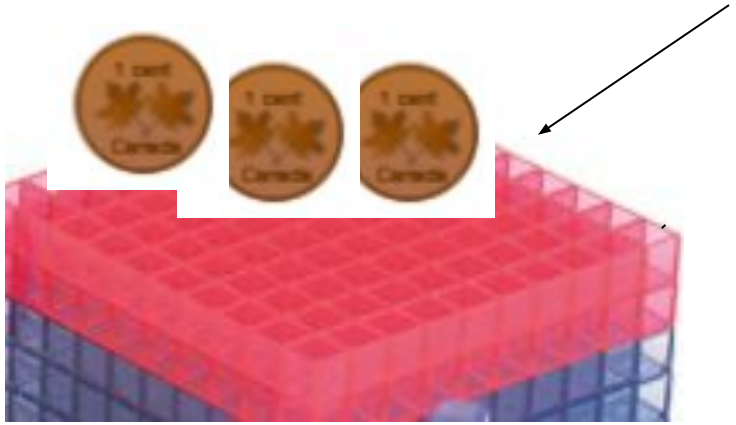
Representing Amounts

Draw or glue coins to represent the amount shown.

103¢	
122¢	
137¢	
155¢	
174¢	
181¢	
196¢	

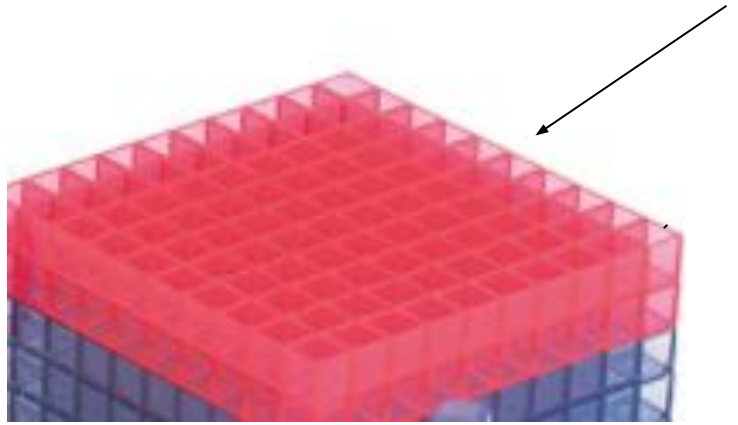
Money APP

Starting Place Value
Show me 103 cents



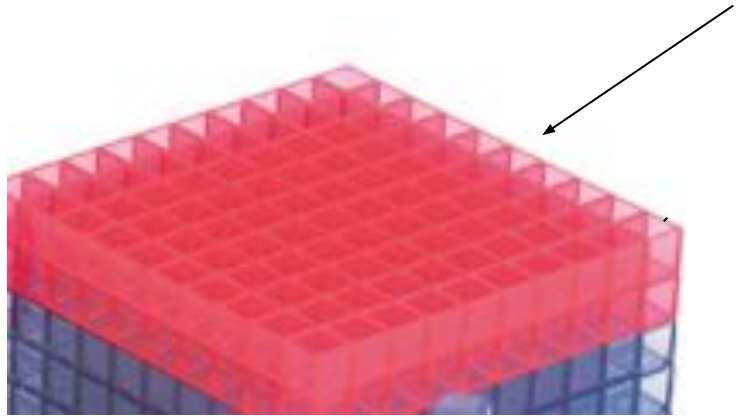
Starting Place Value

Show me $1.26

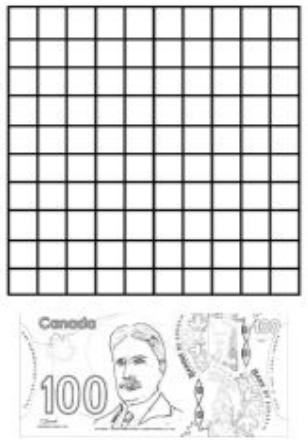
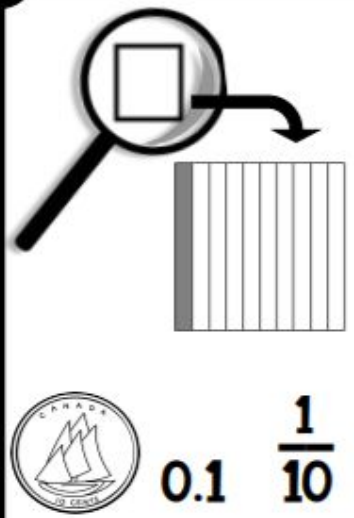
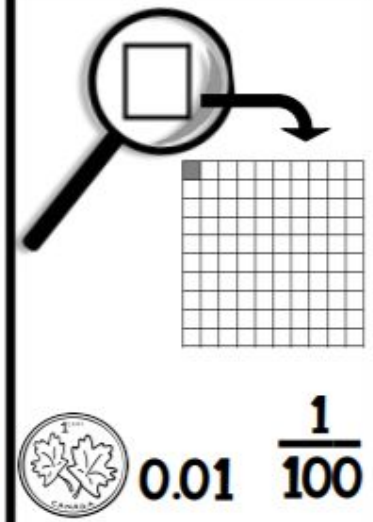


Starting Place Value

Show me 196 cents or $1.96



Billions			Millions			Thousands					
H	T	O	H	T	O	H	T	O	H	T	O
						4	4	5	1	3	7
2	3	4	5	6	7	8	9	0	1	4	5

Hundreds	Tens	Units	Tenths	Hundredths
 100.00	 10.00	 1.00	 0.1 $\frac{1}{10}$	 0.01 $\frac{1}{100}$

So what does the research tell us about fractions and Number lines ?

An opportunity for a re-consider!

Research on Number Lines

"Experience moving physically along numbers sequenced in a line helps children develop a 'mental number line'—a spatial representation of quantity that helps them reason about which is more and how much more."YM

Dr. Nicki Newton, Counting is More Than 1, 2, 3, 2022

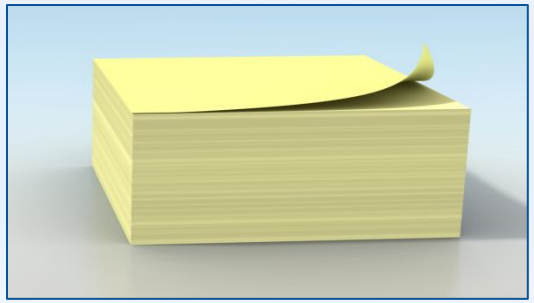
Number Line

Students Internalize Concepts (near, far, more, less, before, after, one more or two more)

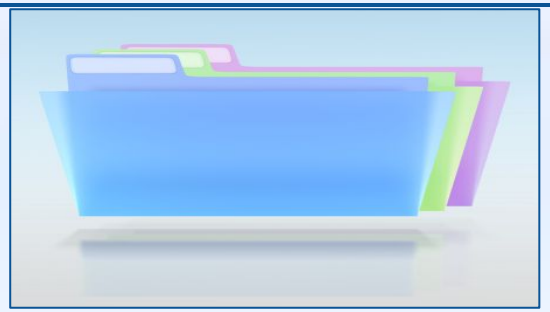
Students can compare numbers

Teachers must build vocabulary throughout the activity

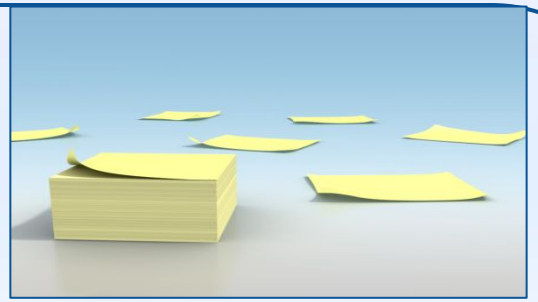
Dr. Nicki Newton, Counting is More Than 1, 2, 3, 2022



- Fractions involve difficult-to-learn and difficult-to-teach concepts that present ongoing pedagogical challenges to the mathematics education community.



- These difficulties begin early in the primary years (Empson & Levi, 2011; Moss & Case, 1999) and persist through middle school (Armstrong and Larson, 1995; Kamii and Clark, 1995), then into secondary and even tertiary education (see Orpwood, Schollen, Leek, Marinelli-Henriques, & Assiri, 2011).



- The challenges and misunderstandings students face in understanding fractions (Gould, Outhred, & Mitchelmore, 2006; Hiebert 1988; NAEP, 2005) persist into adult life and pose problems in such wide-ranging fields as medicine and health care, construction and computer programming.

So what did the research tell us?

1

Understanding:
the roles and relationships between the numbers

1/3 > 1/2

2

Moving to Algorithms too quickly:
models removed quickly, symbolic terminology, procedural

3

Inconsistent Language

"2 over 5" or "2 out of 5" is actually "two fifths"

4

Lack of Connection to Previous Learning:
assume it is obvious a child knows what a fraction represents; real world connections

5

Unit Fraction Glance:

insufficient time devoted to the concept of the "unit" fraction as opposed to algorithms and mathematical definitions.

6

The 'Why' of the Rules:
Lack of time devoted to why we do what we do;connections to unit fraction.

7

Representing -
Over-emphasis on Part-Whole:
Fractions are not only part-whole, many other representations

8

North-American overuse of Circles:
Circles offer us limited understanding when first learning fractions

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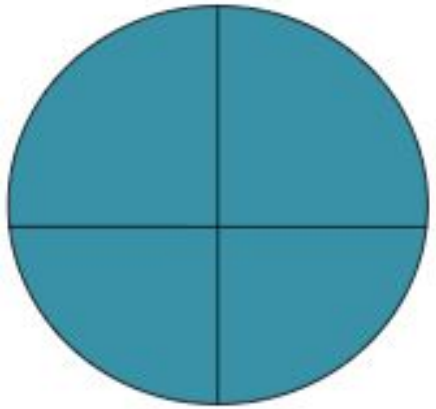
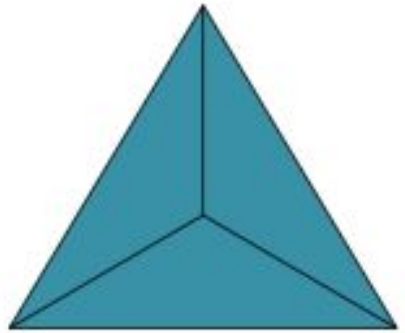
Bruce,C. *Actions to Develop Fraction Understanding*

Avoid this ‘count and fill in’ type of worksheet soon after we talk about a unit fraction.

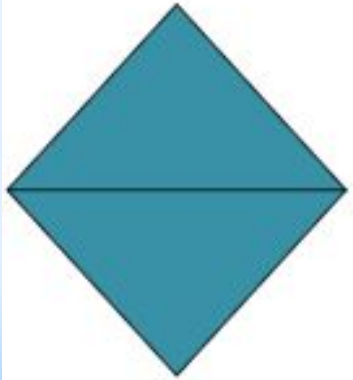
Find the unit fraction for these diagrams



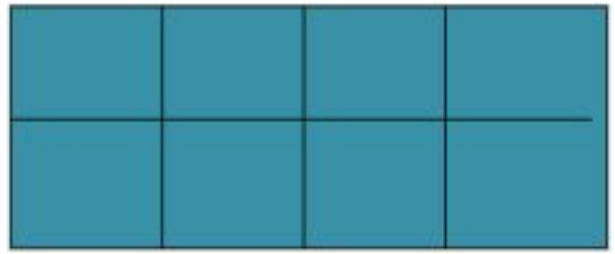
$\frac{1}{\square}$



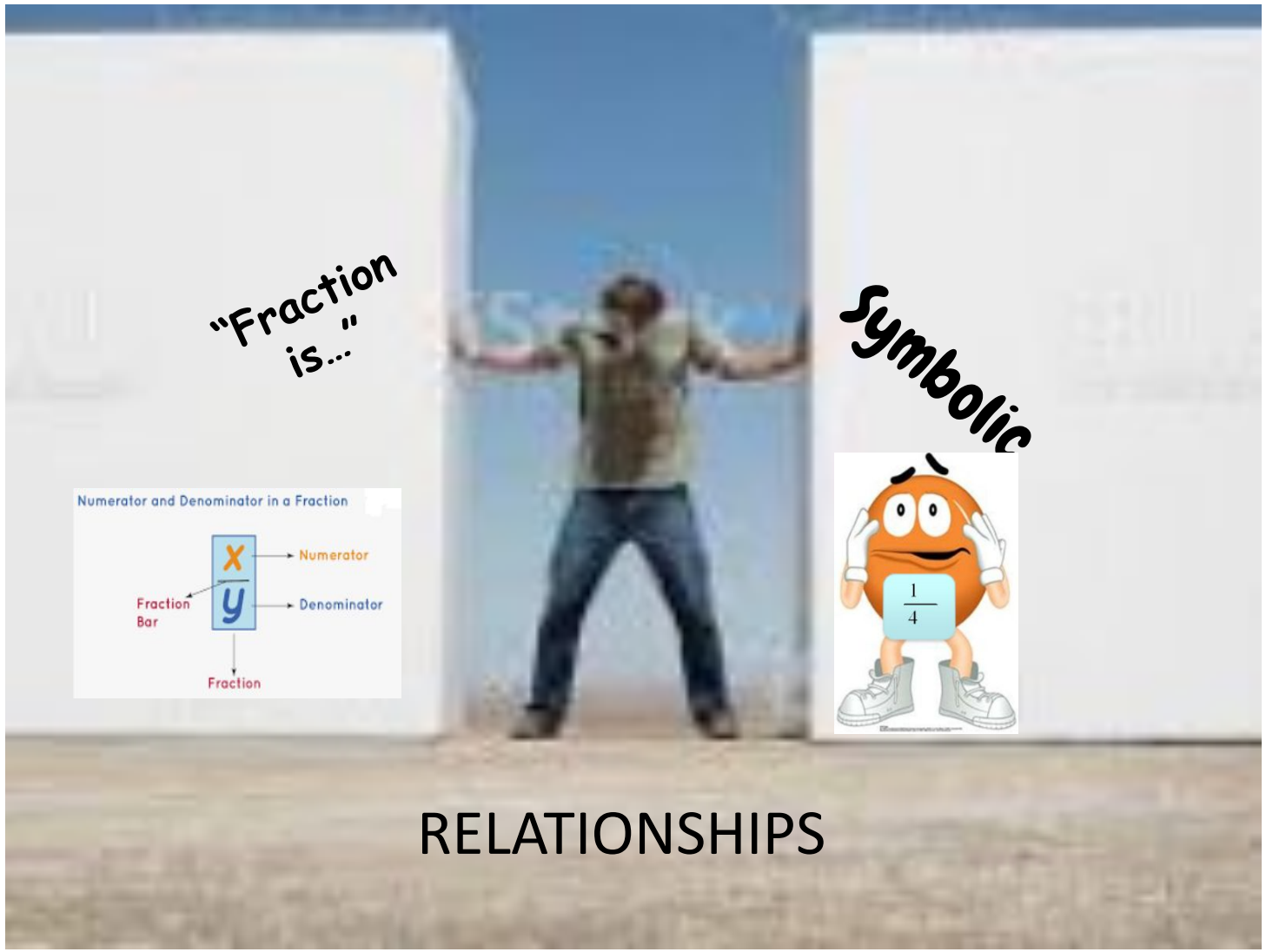
$\frac{1}{\square}$



$\frac{1}{\square}$



$\frac{1}{\square}$



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WHAT IS A FRACTION?

A fraction is a number which can tell us about the relationship between two quantities.

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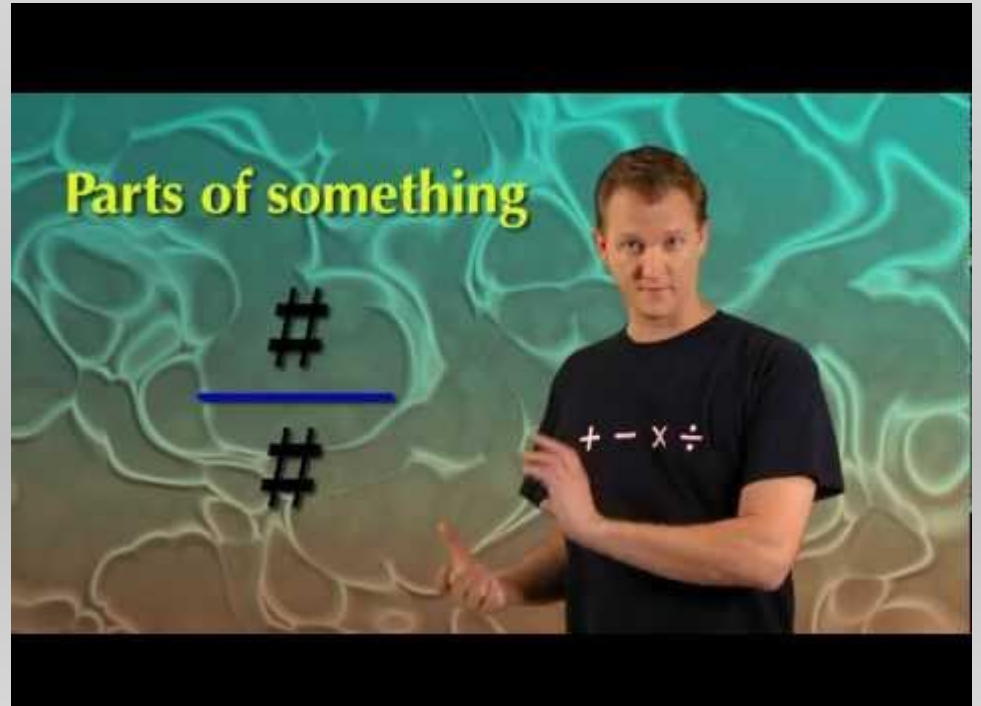
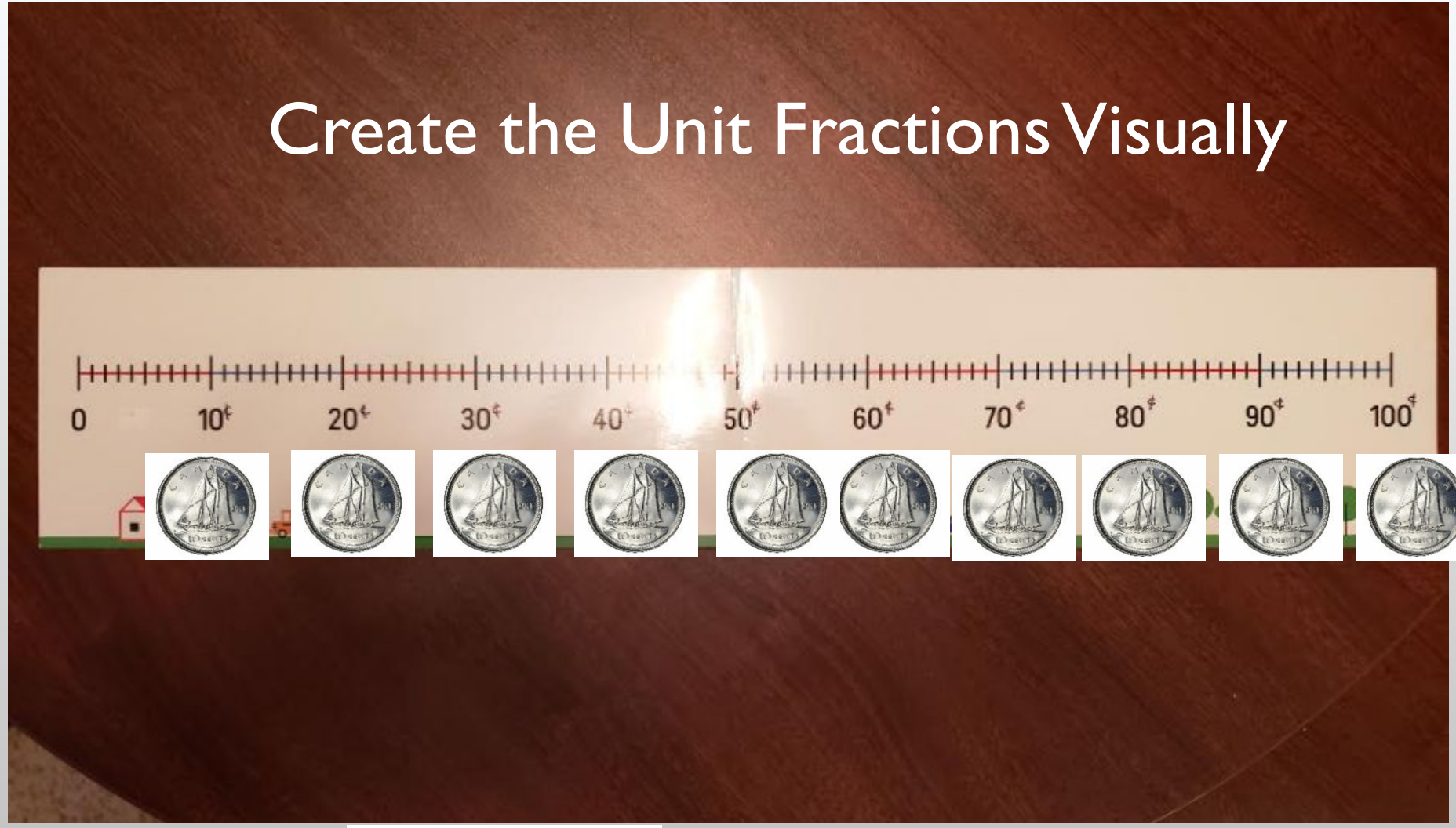
Fractions *can* represent part-to whole relationships



Let’s move to Unit Frac  and Money

Create the Unit Fractions Visually

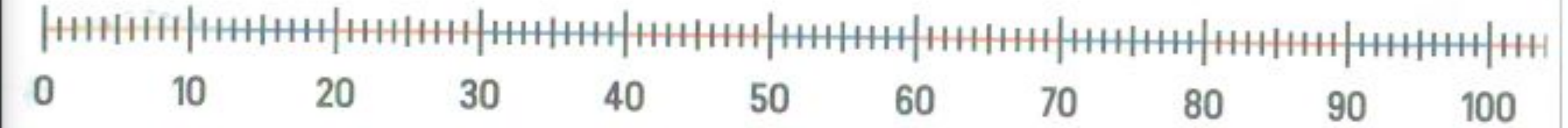


$$\frac{1}{10}$$

Numerator
(1 dime)

Denominator (10 dimes for 1 dollar)

Same coin being represented


0 10 20 30 40 50 60 70 80 90 100


0 10 20 30 40 50 60 70 80 90 100


0 10 20 30 40 50 60 70 80 90 100


0
10¢
1/10
20¢
2/10
30¢
3/10
40¢
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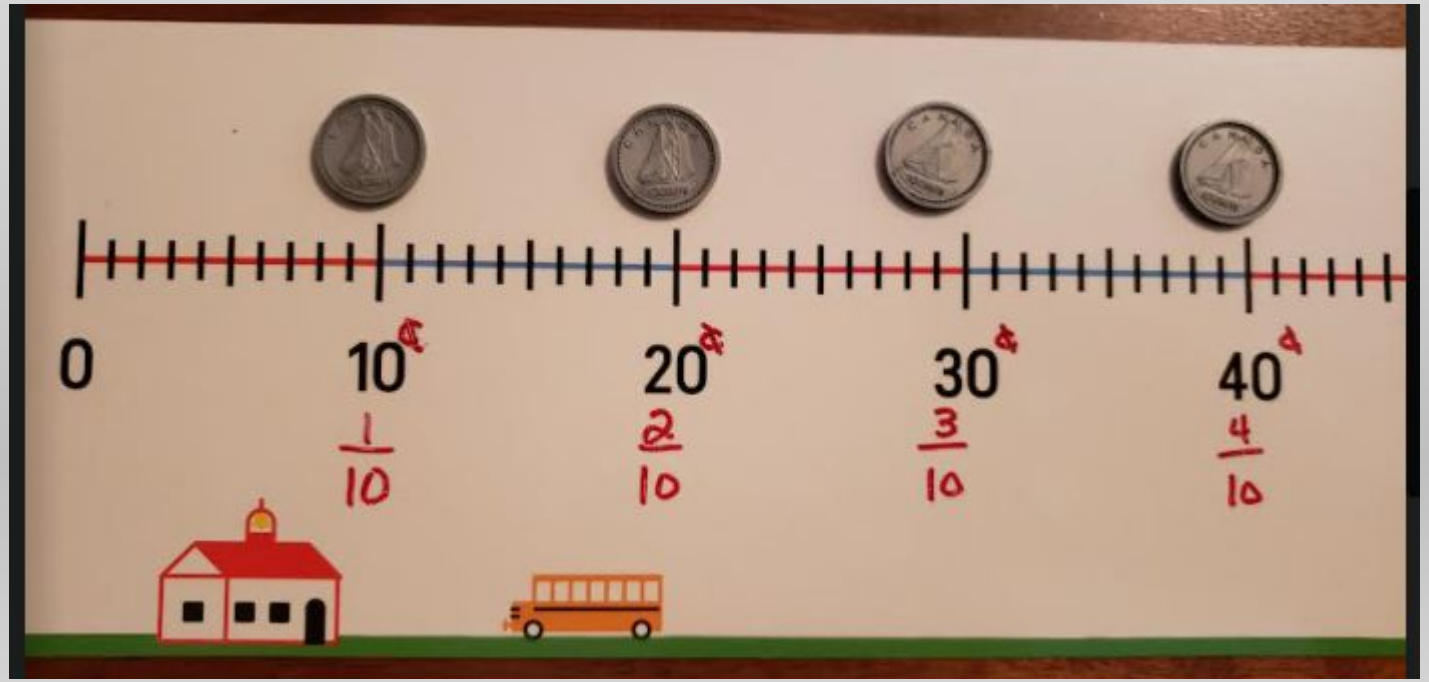
90¢
9/10
100¢
10/10


0
10¢
1/10
20¢
2/10
30¢
3/10
40¢
4/10

Denominator

The denominator says how many equal parts and the quantity being represented

$\frac{1}{2}$

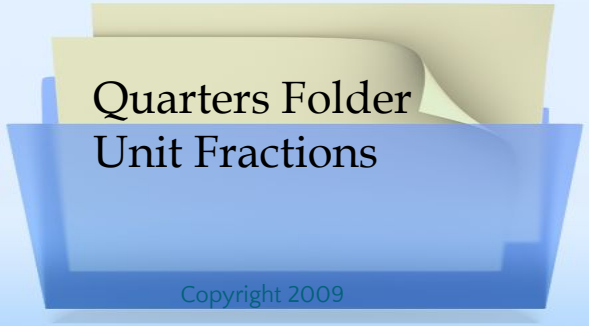
2 50 ¢ coins is 1 dollar or 100 ¢

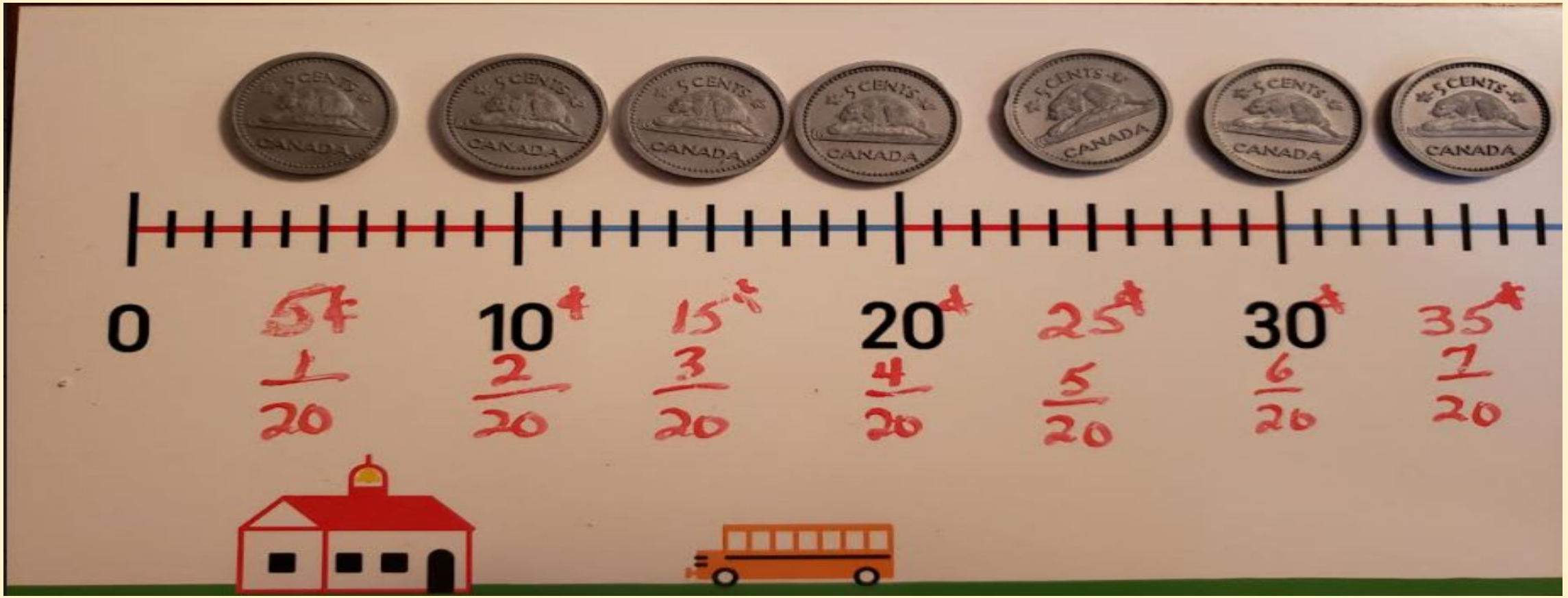
Same coin being represented

•Using more money unit fractions

Use the top line to complete the nickels unit fractions

Use the bottom line to complete the quarters unit fractions




5 CENTS
CANADA
0
5¢
1/20
10¢
2/20
15¢
3/20
20¢
4/20
25¢
5/20
30¢
6/20
35¢
7/20


0
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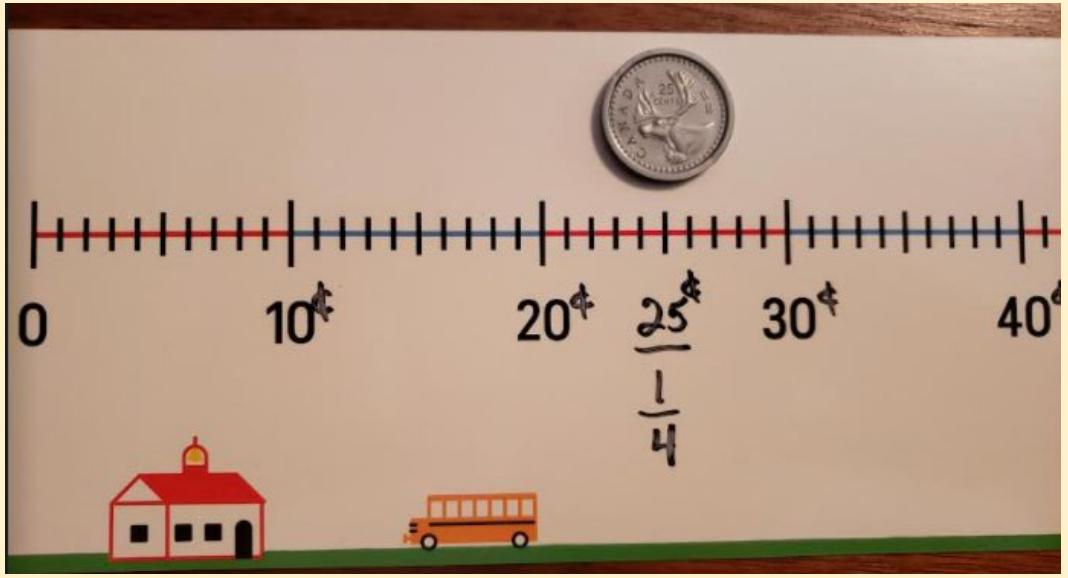
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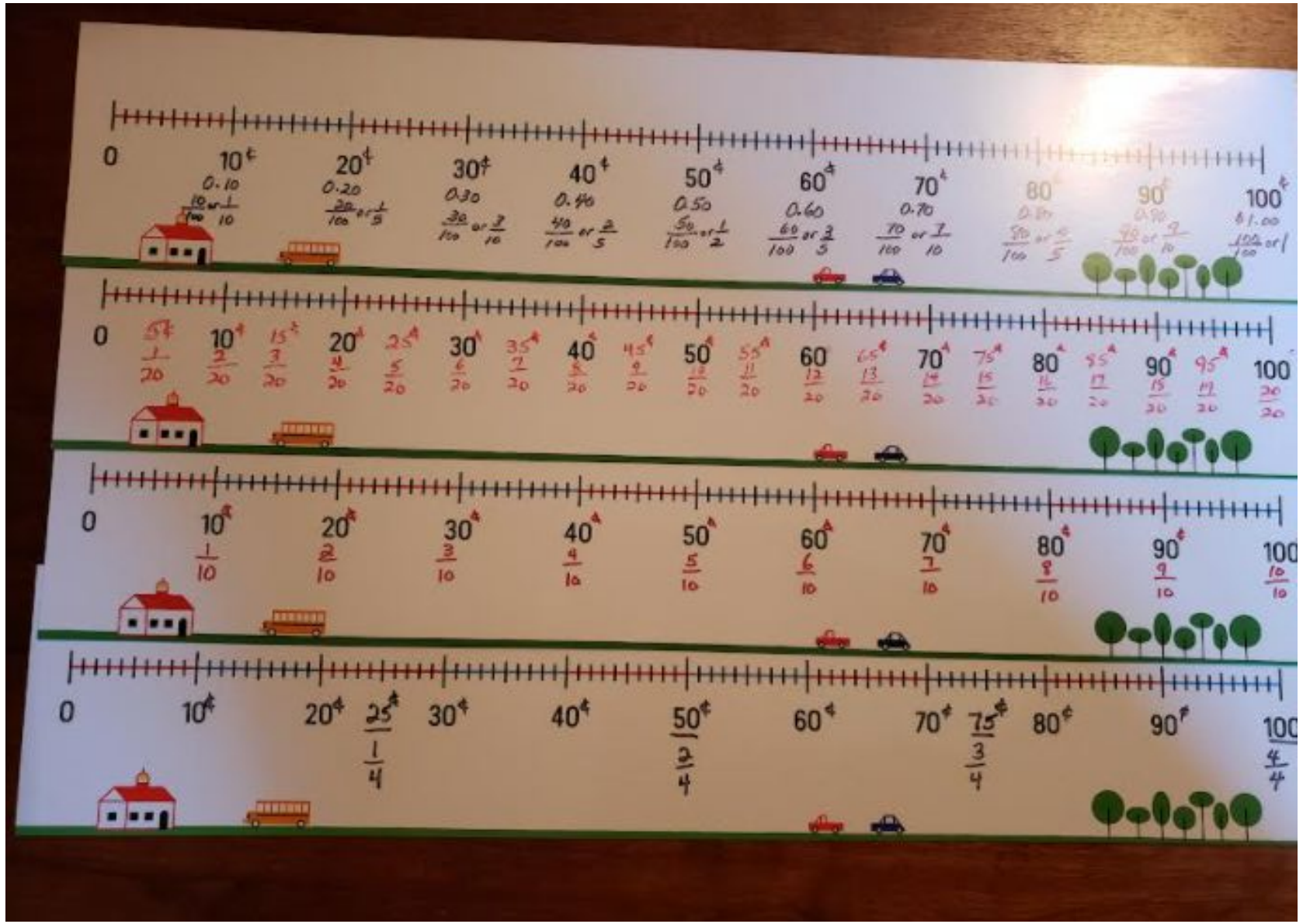
0
10¢ 0.10 10/100 or 1/10
20¢ 0.20 20/100 or 1/5
30¢ 0.30 30/100 or 3/10
40¢ 0.40 40/100 or 2/5
50¢ 0.50 50/100 or 1/2
60¢ 0.60 60/100 or 3/5
70¢ 0.70 70/100 or 7/10
80¢
90¢
100¢ $1.00 100/100 or 1
0
5¢ 1/20
10¢ 2/20
15¢ 3/20
20¢ 4/20
25¢ 5/20
30¢ 6/20
35¢ 7/20
40¢ 8/20
45¢ 9/20
50¢ 10/20
55¢ 11/20
60 12/20
65¢ 13/20
70¢ 14/20
75¢ 15/20
80¢ 16/20
85¢ 17/20
90¢ 18/20
95¢ 19/20
100 20/20
0
10¢ 1/10
20¢ 2/10
30¢ 3/10
40¢ 4/10
50¢ 5/10
60¢ 6/10
70¢ 7/10
80¢ 8/10
90¢ 9/10
100 10/10
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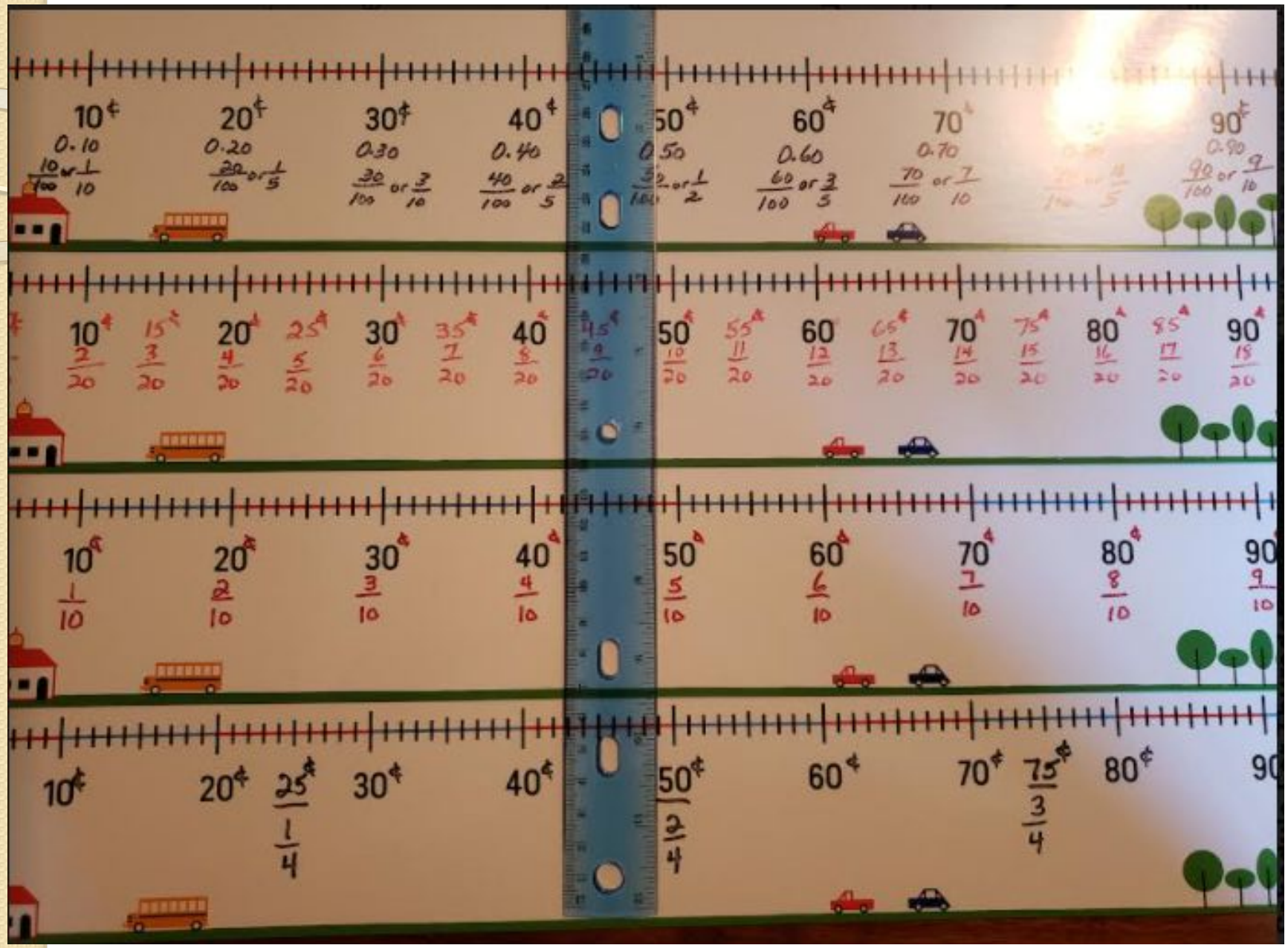
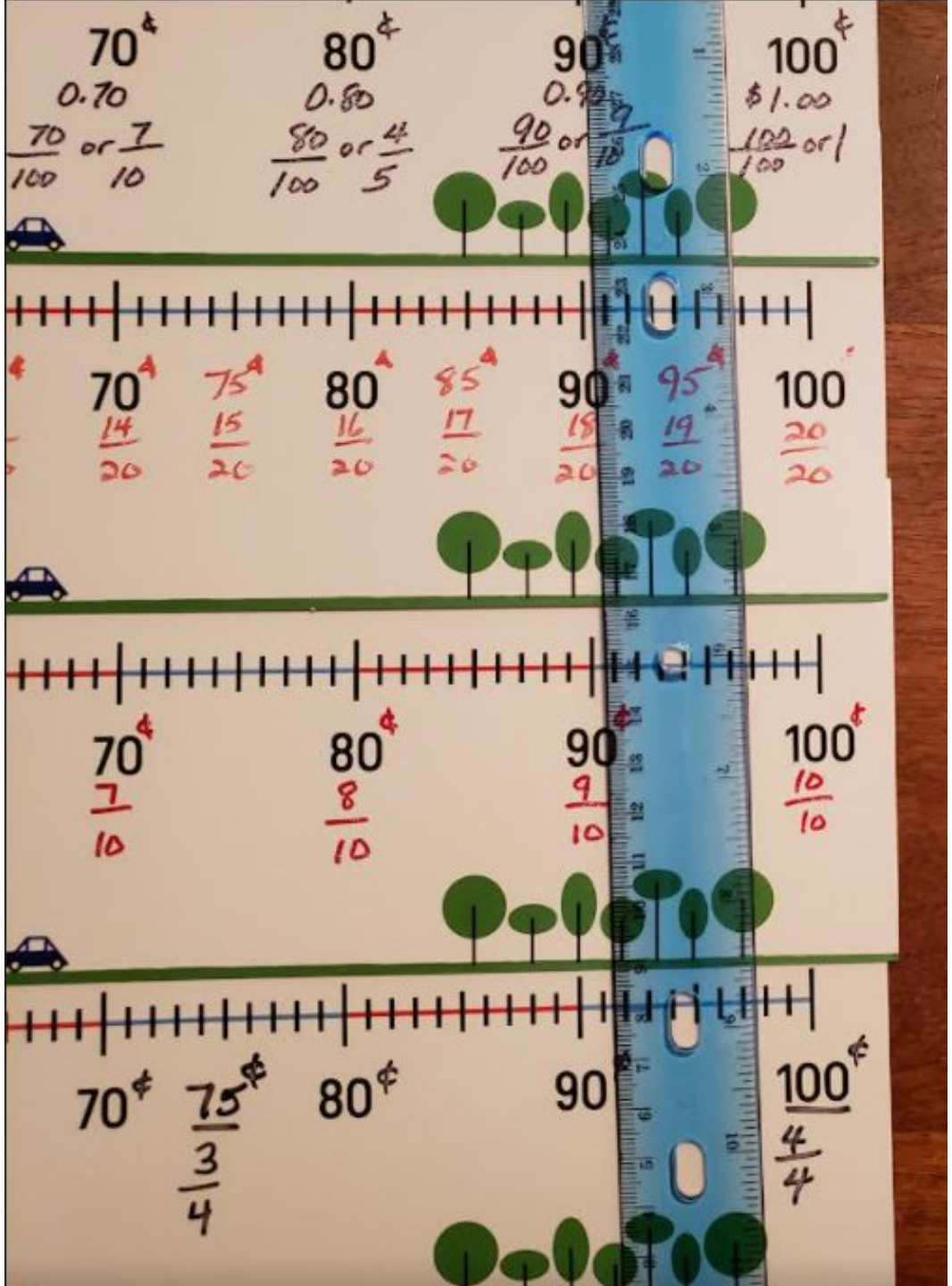
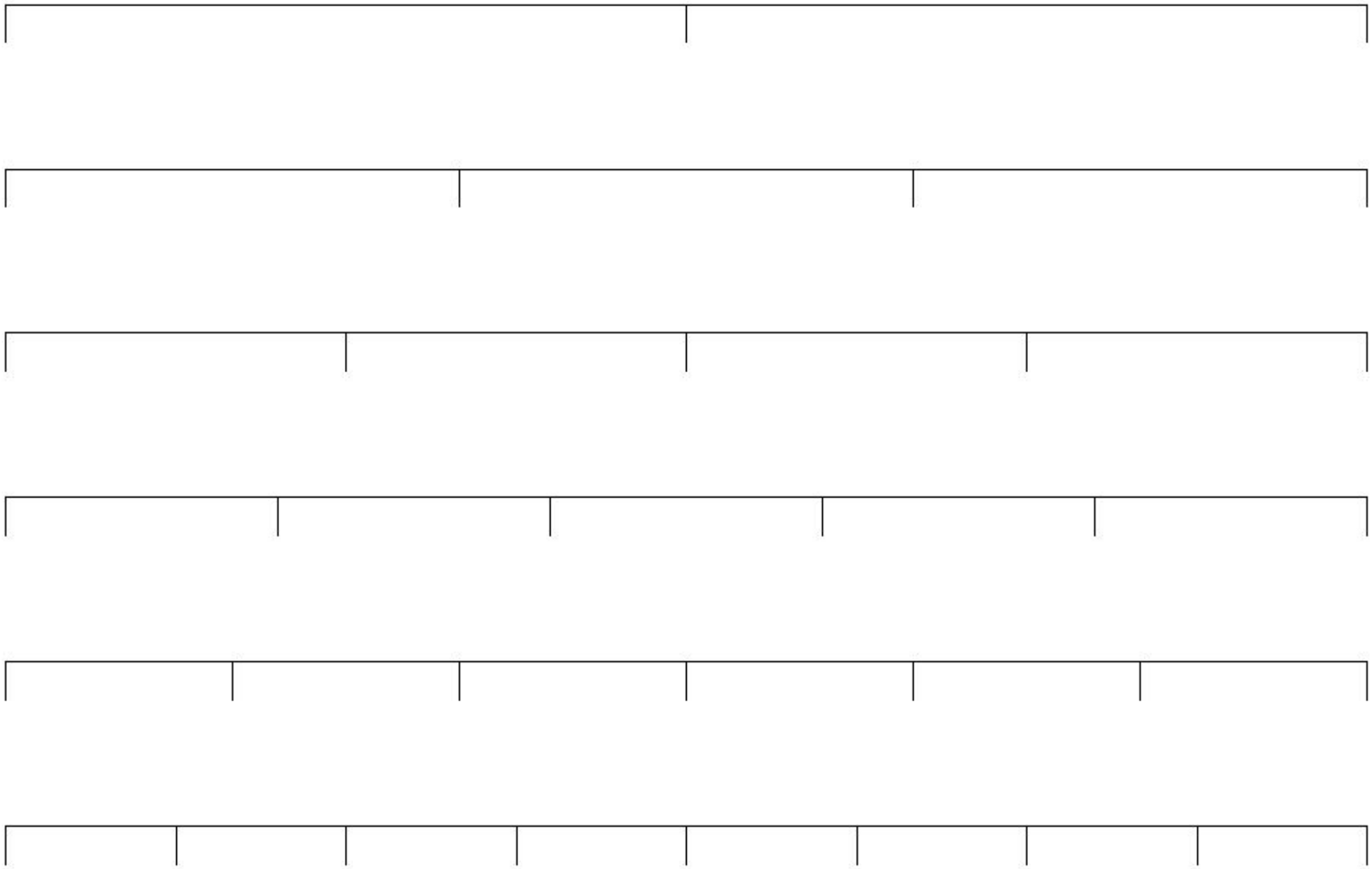
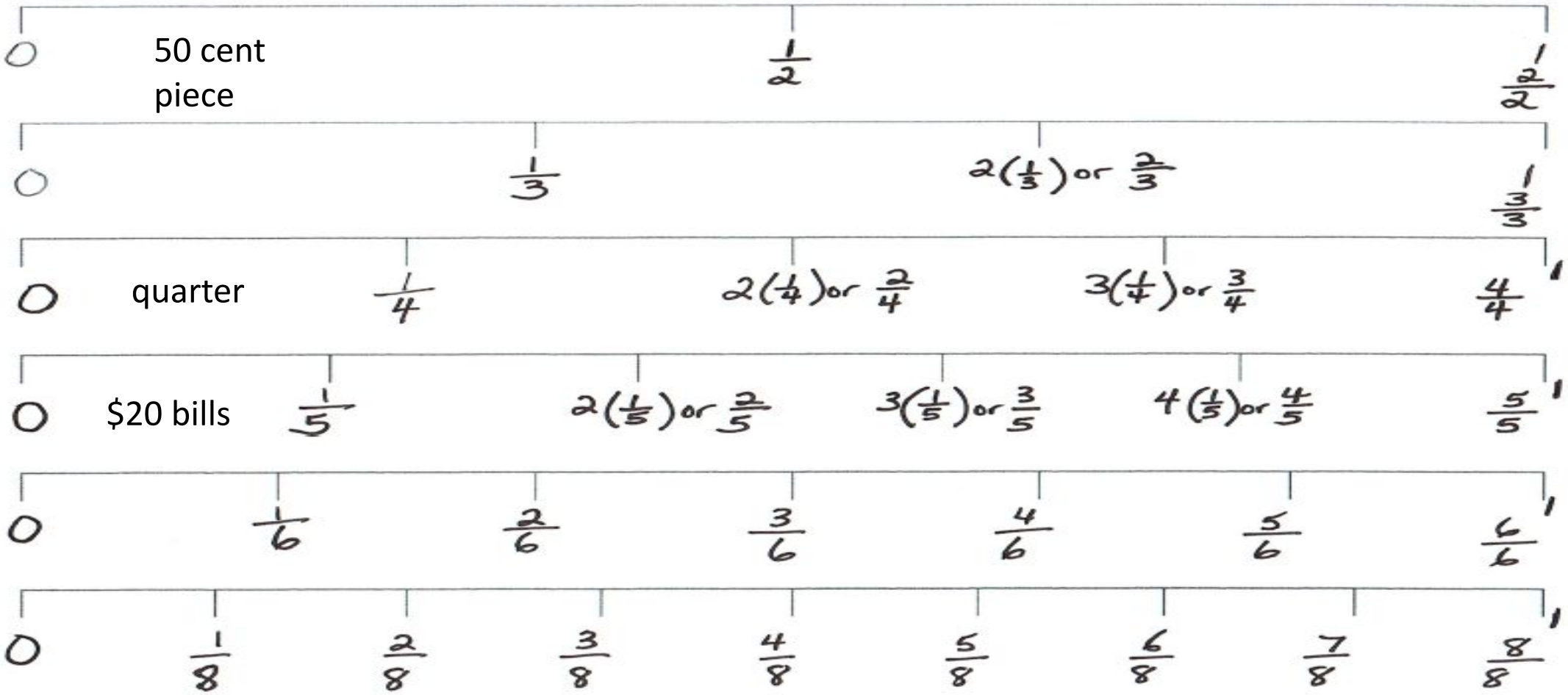
80¢
90¢
100 4/4


10¢ 20¢ 30¢ 40¢ 50¢ 60¢ 70¢ 90¢
0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.90
10/100 or 1/10 20/100 or 1/5 30/100 or 3/10 40/100 or 2/5 50/100 or 1/2 60/100 or 3/5 70/100 or 7/10 90/100 or 9/10
10¢ 15¢ 20¢ 25¢ 30¢ 35¢ 40¢ 45¢ 50¢ 55¢ 60¢ 65¢ 70¢ 75¢ 80¢ 85¢ 90¢
2/20 3/20 4/20 5/20 6/20 7/20 8/20 9/20 10/20 11/20 12/20 13/20 14/20 15/20 16/20 17/20 18/20
10¢ 20¢ 30¢ 40¢ 50¢ 60¢ 70¢ 80¢ 90
1/10 2/10 3/10 4/10 5/10 6/10 7/10 8/10 9/10
10¢ 20¢ 25¢ 30¢ 40¢ 50¢ 60¢ 70¢ 75¢ 80¢ 90
1/4 2/4 3/4

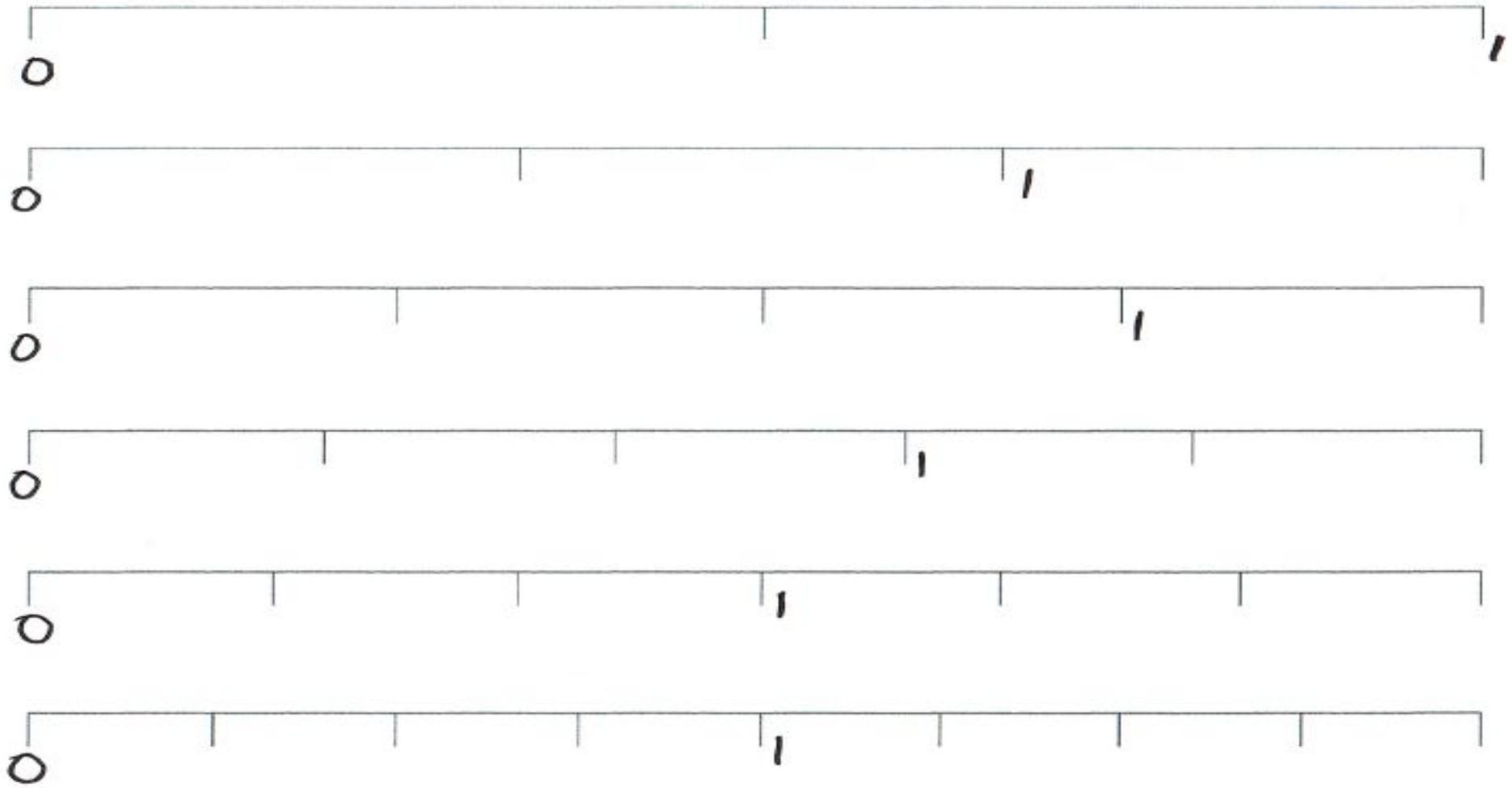

70¢ 0.70 70/100 or 7/10
80¢ 0.80 80/100 or 4/5
90 0.90 90/100 or 9/10
100¢ $1.00 100/100 or 1
70¢ 75¢ 80¢ 85¢ 90 95¢ 100
14/20 15/20 16/20 17/20 18/20 19/20 20/20
70¢ 80¢ 90 100¢
7/10 8/10 9/10 10/10
70¢ 75¢ 80¢ 90 100¢
3/4 4/4



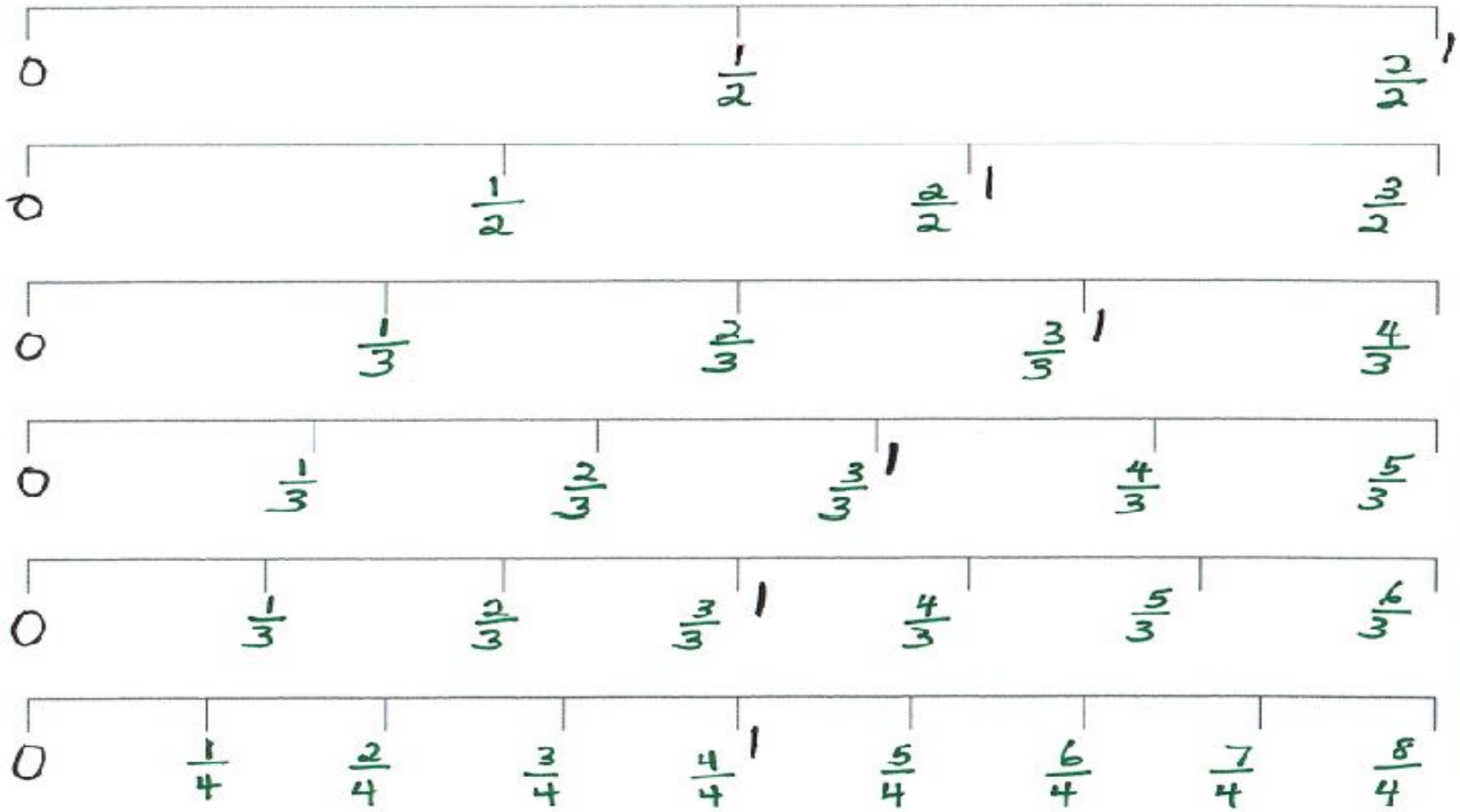


Compare

How can you recognize half?



Surface
Deep
Transfer

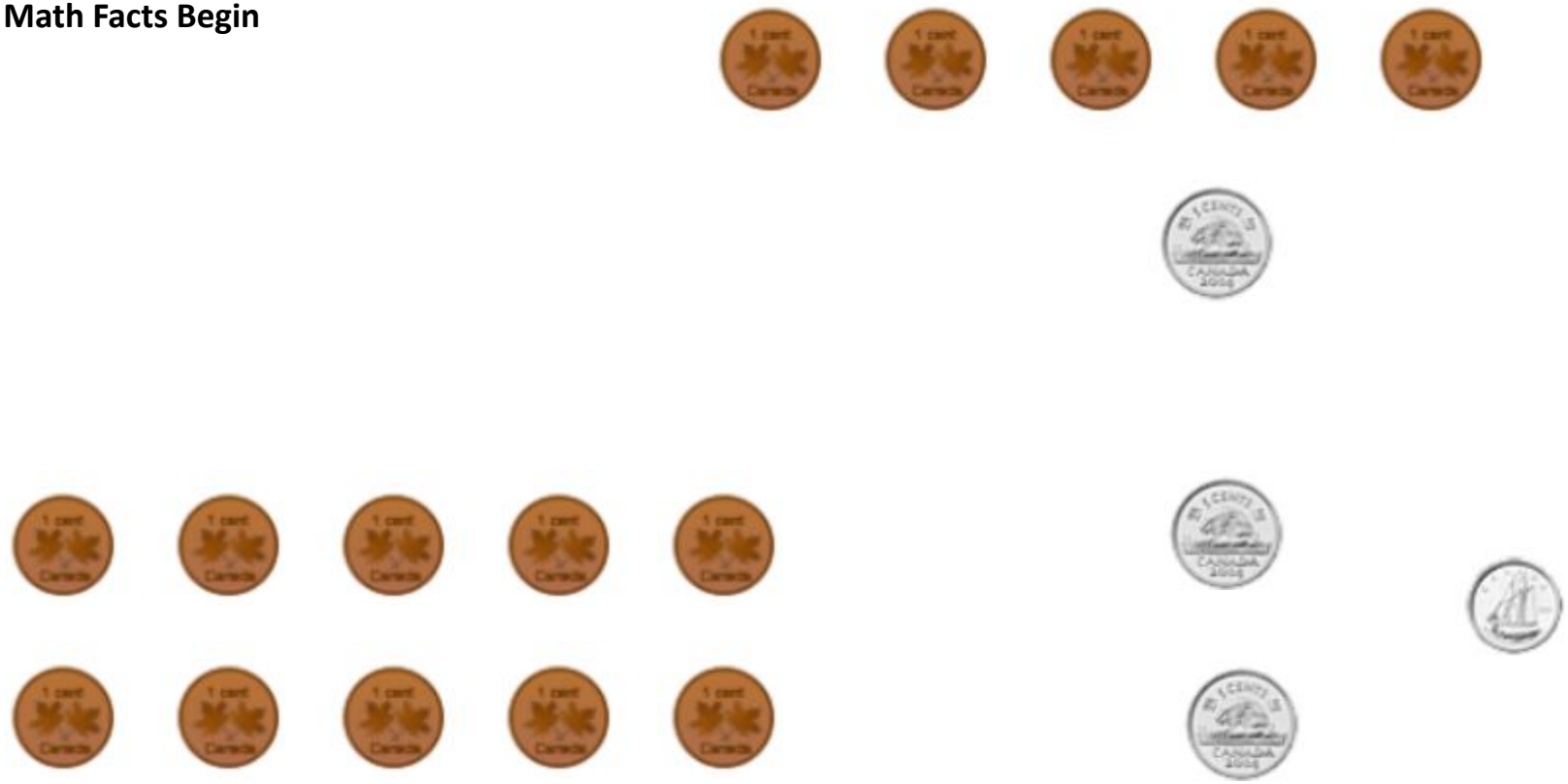


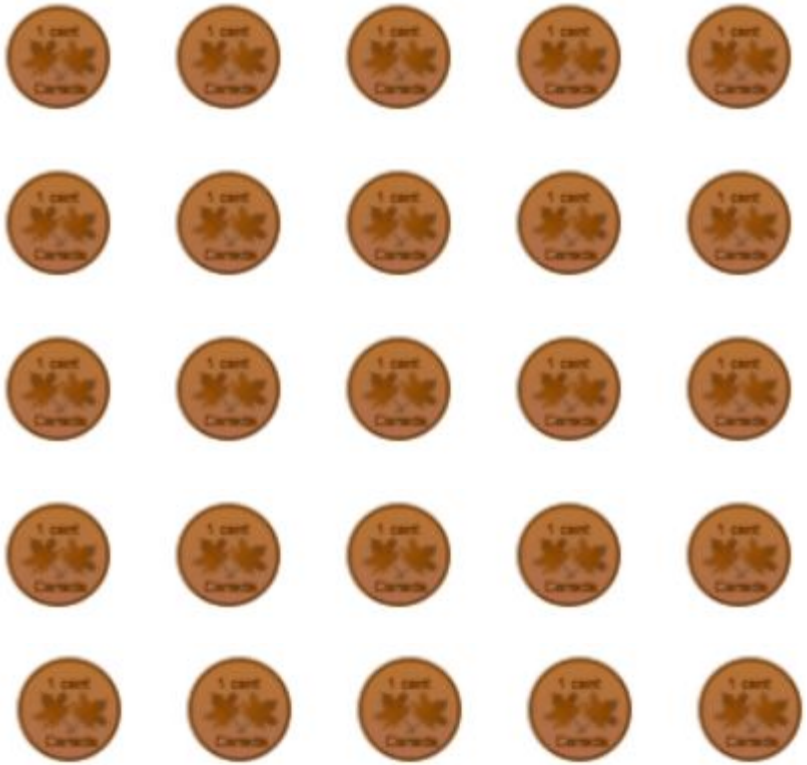
How much is $\frac{3}{2}$?

Money and Math Facts

Conversions Begin With Money

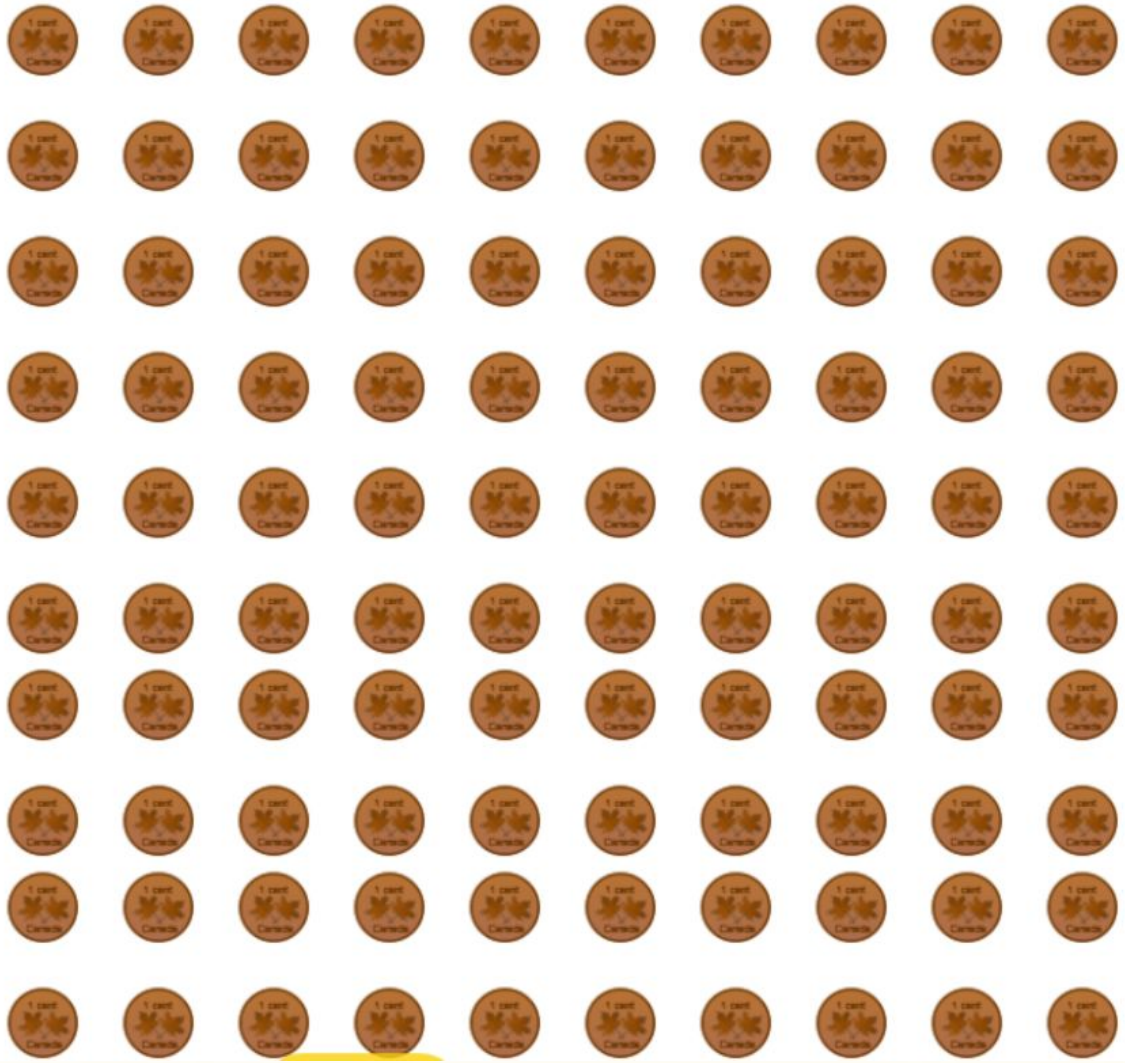
Math Facts Begin





Skip Count
Addition
Multiplication (arrays) - which one’s can we see

Stepping Stones to *UNIT* Fractions



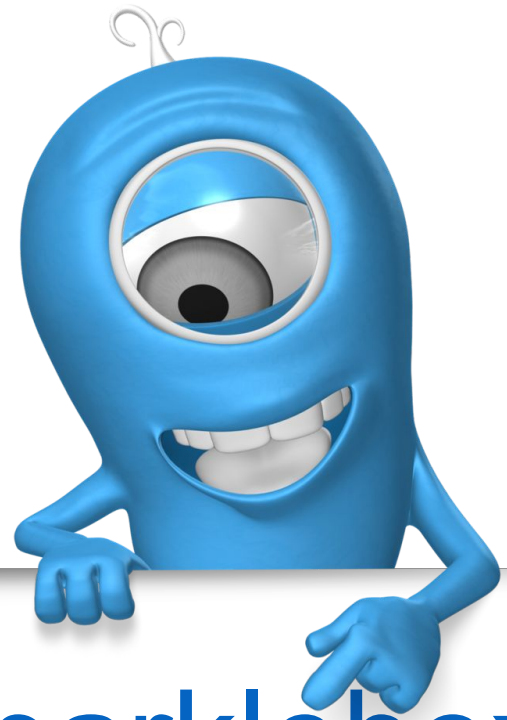
What other representations?

What is this? What is the ‘core’? Extend it - create a new pattern with the same core pattern - create a unique 4 core pattern

four of anything, four of the same,,how much is 4 of the same?

Each dot is 10
Each dot is 100

Learning Trajectories



Sparklebox

Number Lines and resources

Math Games - Fractions

Grade 1 Ontario Math - Fractions

https://cubeforteachers.com/folders/3bJqkwyKXANApxl6s6y8lf3lOYFd2M5slcPu9YB2KBiwZGFjdKAUA5kOphY5NwHM?q=1.B1.6&idx=posts&p=0

Grade 2 Ontario Math - Fractions

https://cubeforteachers.com/folders/PliSotwOSqBY4iodosCiTWSGylnrf1rviPP7JvZXyAqIPNKPo3adGhXXmoRd1NbU?q=2.B1.6&idx=posts&p=0

Grade 3 Ontario Math - Fractions

https://cubeforteachers.com/folders/5wNEuWt6xc8b5csw9hsWuCpYm3bdyOikqek3C0nP6JpDhKJLwlsOVkKfX3ytgWtF?q=3.B1.6&idx=posts&p=0

Grade 4 Ontario Math - Fractions

https://cubeforteachers.com/folders/uQLGiZSylxmhQMNFRnIHxeYeQBptrEma9oyvJpFujtN2L9AsaDsmI3EVrAgcQAGz?q=4.B1.4&idx=posts&p=0

Sources:

Bruce, C., Chang, D., Flynn, T. *Foundations to Learning and Teaching Fractions: Addition and Subtraction - Literature Review.* Trent University. 2013

Bruce, C et al. Actions to Develop Fractions Understanding as contained in *Paying Attention to Mathematics Education.* http://www.edugains.ca/resourcesMath/CE/LessonsSupports/Fractions/SupportDocs/ActionstoDevelopUnderstandingFractions.pdf

__. Institute of Educational Science. *IES - Developing Effective Fractions Instruction for Kindergarten Through Grade 8.* 2010 https://www.readkong.com/tmp/developing-effective-fractions-instruction-for-kindergarten-7236374.pdf

Greenberg, D. Funny and Fabulous Fraction Stories. Scholastic Education

Ways We Use Fractions. Edugains Teacher Support: http://www.edugains.ca/resourcesDP/Resources/PlanningSupports/mathforTeachingWaysWeUseFractions.pdf

Scholastic Educational Supplies

Thank you!

Chris Zarski
czarski@carcpd.ab.ca

Where are the files going to be?

Grade 3 Folder